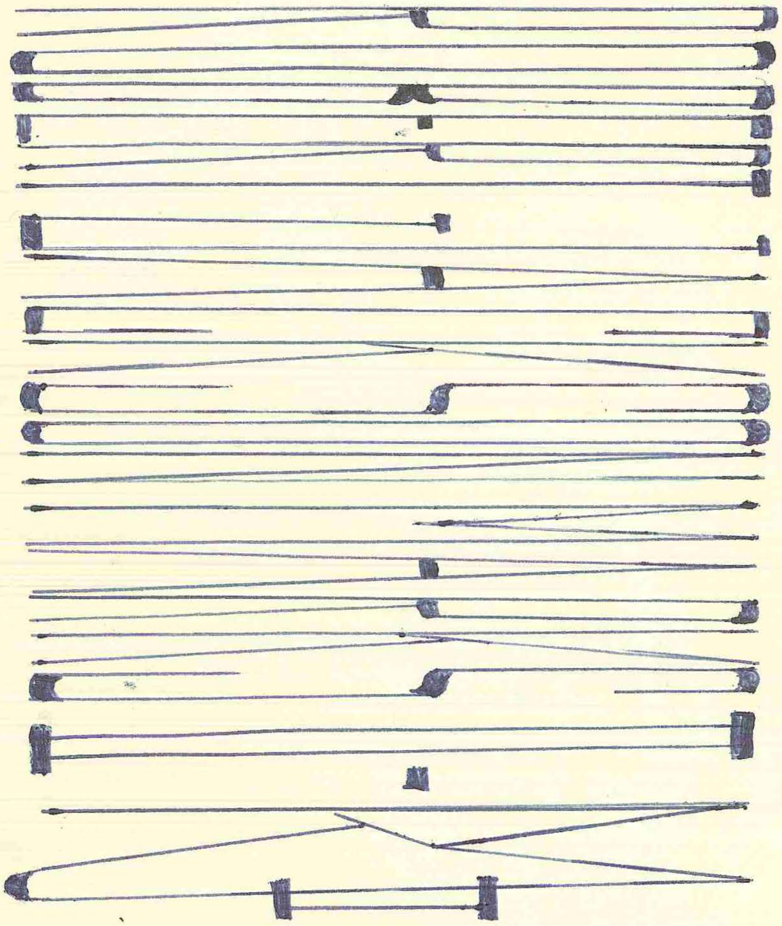
Reliability

R.J. Marks II Class Notes

Rock Island Arsenal (1973)



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PROBABILITY (SECTION II)

I. PROBABILITY DEFINITIONS

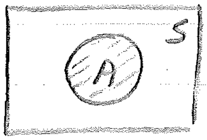
A. CLASSICAL OR APRIORI : IF, IN $a+b$ EXPERIMENTS, AN EVENT 'A' CAN OCCUR IN a WAYS, THEN THE PROBABILITY OF EVENT a OCCURING, $P[A]$, IS

$$P[A] = \frac{a}{a+b}$$

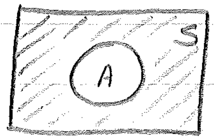
B. RELATIVE FREQUENCY OR POSTERIORI APPROACH: IF m SUCCESSES OCCUR FROM n EVENTS, WE APPROXIMATE

$$P[A] = \frac{m}{n}$$

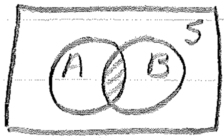
II. VENN DIAGRAMS



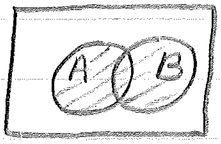
A



$\bar{A}$



(AND)
$A \cap B$



(OR)
$A \cup B$

III. BASIC PROBABILITY LAWS

1. $P[\bar{A}] + P[A] = 1$
2. $P[A \cup B] = P[A] + P[B] - P[A \cap B]$

FOR MUTUALLY EXCLUSIVE EVENTS:

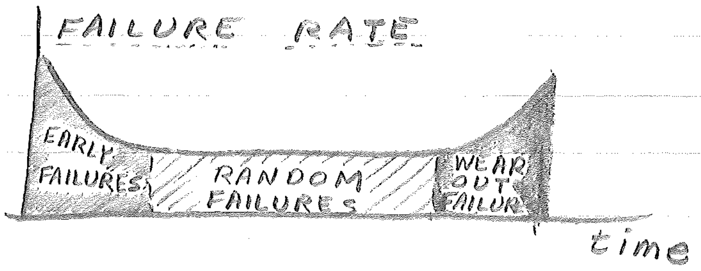
$$P[A \cup B] = P[A] + P[B]$$

3. $P[A \cap B] = P[A]P[B/A] = P[B]P[A/B]$

FOR INDEPENDENT EVENTS

$$P[A \cap B] = P[A]P[B]$$

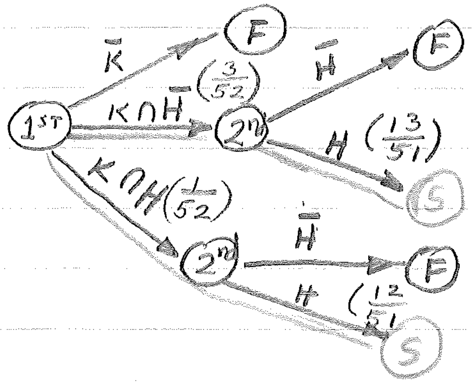
BATHTUB CURVE



TREE DIAGRAM

ILLUSTRATING P[DRAWING A KING AND A HEART FROM 52 CARD DECK]

$$= \frac{3}{52}\left(\frac{13}{51}\right) + \frac{1}{52}\left(\frac{12}{51}\right)$$



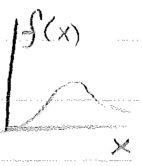
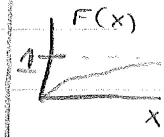
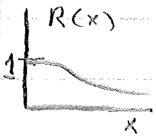
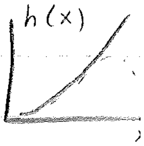
{ F → FAILURE
S → SUCCESS }

PROBABILITY FUNCTIONS: MATHEMATICAL EXPRESSIONS USEFUL IN DESCRIBING THE LIFE CHARACTERISTICS OF A POPULATION OF ITEMS.

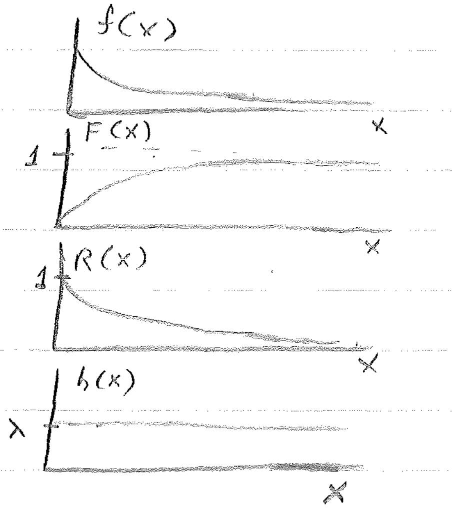
RANDOM VARIABLE

1. DISCRETE: FAILURES THAT OCCUR IN GIVEN TIME INTERVAL
2. CONTINUOUS: TIME AT WHICH ITEM FAILS

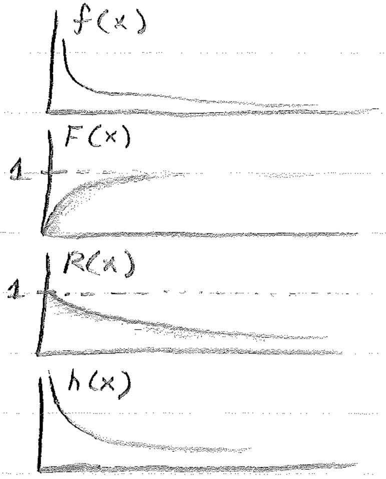
COMMONLY USED

FUNCTION	SYMBOL	DESCRIPTION	RESTRICTION	RELATIONSHIP	ILLUS.
DENSITY	$f(x)$	FAILURE RATE AT TIME X PER ITEM WHICH WAS ALIVE AT TIME 0.	$0 \le f(x)$ $\int_{-\infty}^{\infty} f(x)dx = 1$	$f(x) = \frac{d}{dx}F(x)$	
DISTRIBUTION	$F(x)$	PROPORTION OF ITEMS FAILED AT OR PRIOR TO TIME X. FOR ONE ITEM $P[X \le x] = F(x)$	$0 \le F(x) \le 1$ $R(x)$ IS MONOTONIC INCREAS.	$F(x) = \int_{-\infty}^{x} f(x)dx$	
RELIABILITY	$R(X)$	PROPORTION OF ITEMS SURVIVING AT TIME X FOR ONE ITEM. $P[X > x] = R(x)$	$0 \le R(x) \le 1$ $R(x)$ IS MONOTONIC DECREASING	$R(x) = 1 - F(x)$ $R(X) = e^{-\int_{-\infty}^{x} h(x)dx}$	
HAZARD	$h(x)$	FAILURE RATE AT TIME X PER ITEM WHICH WAS ALIVE IN TIME X	$0 \le h(x)$	$h(x) = \frac{f(x)}{R(x)}$	

1. FOR $h(x) = \text{CONSTANT} = \lambda$



2. FOR $h(x)$ MONOTONICALLY DECREASING

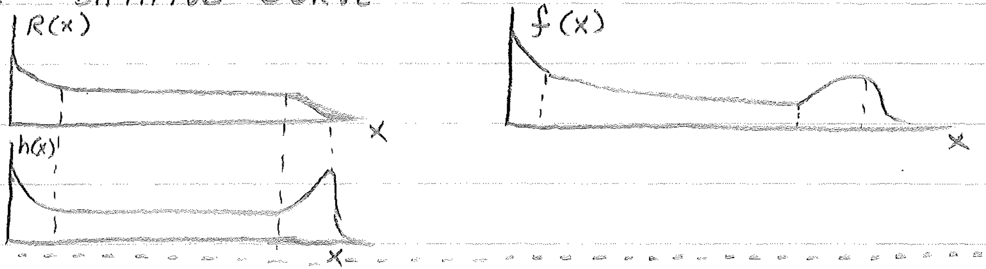


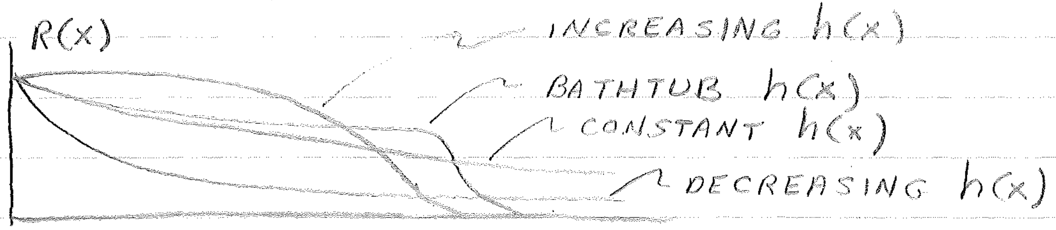
THE BATHTUB CURE IS A HAZARD CURVE, $h(x)$

$h(x)$ AND $f(x)$ READ IN FAILURES/TIME

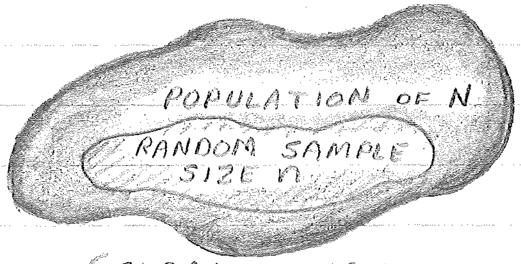
$R(x)$ $[F(x)]$ READ IN SUCCESSES [FAILURES]

3. FOR BATHTUB CURVE





(PREDICTING)
I. IN OBSERVING THE LIFE CHARACTERISTICS OF A POPULATION OF ITEMS, WE ARE INTERESTED IN QUANTITATIVE DESCRIPTION OF THE POPULATION



PARAMETERS DESCRIBE POPULATION
STATISTICS DESCRIBE SAMPLE

SOME PARAMETERS OF INTEREST
- CENTRAL TENDENCY (AVERAGE)
- DISPERSION (VARIANCE)
- DISTRIBUTION TYPE

MEASURES OF CENTRAL TENDENCY

1. ARITHMETIC MEAN: $\bar{X} = \frac{1}{n} \sum_{n} X_i$
2. MEDIAN: THAT TIME X AT WHICH 50% OF THE ITEMS HAVE FAILED
3. MODE: THE MOST FREQUENTLY OCCURING FAILURE TIME

POPULATION MEAN: $\mu = \frac{1}{N} \sum_{N} X_i$

$\bar{X}$ IS A STATISTIC DESCRIBING POPULATION MEAN μ.

MEASURES OF SPREAD

1. RANGE: DIFFERENCE BETWEEN HIGHEST AND LOWEST VALUE.
2. STANDARD DEVIATION:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}} = \frac{1}{N}\sqrt{N\sum_{i=1}^{N} X_i - \left(\sum_{i=1}^{N} X_i\right)^2} \Bigg\} \text{FOR POPULATION}$$

$$\left.\begin{aligned} s &= \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}} \\ &= \sqrt{\frac{1}{n(n-1)}\left[n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2\right]} \end{aligned}\right\} \text{FOR SAMPLE}$$

$V(x) = \sigma^2$

S IS A STATISTIC DESCRIBING POPULATION MEAN σ

EXPECTED VALUE

$E(X) = \mu$

CLASS
PROBLEM: GIVEN A LOT OF TEN GENERATORS, 3 OF WHICH ARE KNOWN DEFECTIVE, FIND THE PROBABILITY OF SELECTING A SAMPLE OF 3 GOOD GENERATORS. (SAMPLE WITHOUT REPLACEMENT.)

ANSWER:

$$\begin{aligned} P[A] &= P[\text{CHOOSING 3 GOOD GENERATORS}] \\ &= P[A_1]\,P[A_2/A_1]\,P[A_3/A_2 \cap A_1] \\ &= \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \\ &= \frac{7}{24} \\ &= 0.2916666667 \end{aligned}$$

DISCRETE DISTRIBUTIONS

1. BINOMIAL DISTRIBUTIONS

p: P[ITEM FAILS ON A SINGLE TRIAL]

q: P[" SURVIVES " " " "]

$\Rightarrow p + q = 1$

FOR n TRIALS, $(p+q)^n = 1$

BINOMIAL EQUATION:

$$P[X = x] = \frac{n!}{x!\,(n-x)!}\, p^x q^{n-x}$$

$$= \binom{n}{x} p^x q^{n-x}$$

$$P[X \le x] = \sum_{m=0}^{x} \binom{n}{m} p^m q^{n-m}$$

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5. FIND $P[X \ge 1]$ WITH $n = 3$, $p = 0.8$

$$P[X \ge 1] = \sum_{x=1}^{3} \binom{n}{x} p^x q^{n-x}$$

$$= \frac{3!}{(2!)(1!)} (0.8)^1 (0.2)^2 + \frac{3!}{(1!)(2!)} (0.8)^2 (0.2)^1 + \frac{3!}{(0!)(3!)} (0.8)^3 (0.2)^0 = 0.992$$

NOTE:

$$P[X \ge 1] = P[X \ne 0]$$

FOR SAMPLES FROM BINOMIAL POPULATION,

$\sigma^2 = npq.$

$\mu = Np$ [EXPECTED VALUE]

POISSON DISTRIBUTION

CHARACTERISTICS

1. HAZARD (OR FAILURE) RATE, λ, MUST BE CONSTANT
2. AN INTERVAL OF INTEREST, X
3. $\underline{K}$ = # OF FAILURES YET TO OCCUR
4. k = # OF FAILURES OF INTEREST

$$P[\underline{K} = k] = \frac{(\lambda X)^k e^{-\lambda x}}{k!}$$

EXAMPLE:

THREE ITEMS ARE TO BE TESTED FOR 10 HRS., EACH WITH REPLACEMENT. EACH* ITEM HAS A CONSTANT FAILURE RATE OF $\lambda = 0.1$ FAILURES/HR. WHAT IS P[2 FAILURES]:

$k = 2$, $\lambda_T^* = 0.03$, $X = 10$

$$P[\underline{K} = 3] = \frac{(0.03 \times 10)^2 \ e^{-(0.03)(10)}}{2!}$$

$$= \frac{(0.9) \ e^{-0.3}}{2}$$

$$= 0.033337$$

NOTE THAT 0.03 FAILURES ARE EXPECTED.

EXAMPLE:

FOR SAME PROBLEM, FOR JUST ONE ITEM

$$P[K=0] = \frac{(\lambda k)^0}{0!} e^{-0.1} = 0.90984$$

THIS IS THE RELIABILITY OF THE ITEM FOR 10 HRS.

EXAMPLE:

1000 ROUNDS OF AMMO ARE TO BE FIRED. THE PROBABILITY OF FAILURE OF EACH ROUND IS $0.001 = 10^{-3}$. WHAT IS THE PROBABILITY OF EXACTLY 1 FAILURE.

EMPLOYING BINOMIAL:

$$P[X=1] = \binom{1000}{1} (0.001)^1 (0.999)^{999}$$

$$= 0.36806 \Leftarrow \text{CORRECT}$$

EMPLOYING POISSON:

$$P[K=1] = (\lambda x)^k e^{-\lambda x} / k!$$

$$\lambda = 10^{-3}, \quad x = 10^3$$

$$\Rightarrow P[K=1] = e^{-1} = 0.3678 \Leftarrow \text{APPROXIMATE}$$

MAY EMPLOY POISSON TO APPROXIMATE BINOMIAL WHEN λ IS SMALL IN COMPARISON WITH x (ie $\lambda \ll x$)

CONTINUOUS PROBABILITY FUNCTIONS

A. EXPONENTIAL DISTRIBUTION

ONE PARAMETER : $\theta \Rightarrow$ MTBF (MEAN TIME BETWEEN FAILURE)

$\theta = \frac{1}{\lambda} \ni \lambda =$ FAILURE RATE

1. DENSITY FUNCTION:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} = \frac{1}{\theta} e^{-x/\theta} & : \quad x \geq 0 \\ 0 & : \quad x < 0 \end{cases}$$

2. $$F(x) \triangleq \int_{-\infty}^{x} f(x)dx$$
$$= \lambda \int_{0}^{x} e^{-\lambda x} dx$$
$$= 1 - e^{-\lambda x}$$

3. $$R(x) \triangleq 1 - F(x)$$
$$= e^{-\lambda x}$$ $\Leftarrow$ SAME AS POISSON FOR 0 FAILURES

4. $$h(x) \triangleq f(x)/R(x)$$
$$= \lambda$$ $\Rightarrow$ CONSTANT

MEAN: θ

VARIANCE: $\theta^2 = \sigma^2$

PERSONAL SUPPLEMENT

PROOF: THE MEAN OF AN EXPONENTIAL DENSITY FUNCTION IS NOT EQUAL TO THAT POINT WHERE 50% OF THE AREA LIES ON EITHER SIDE.

$$f(x) = \lambda e^{-\lambda x} \mu(x)$$

THE MEAN OF $f(x)$ IS

$$\mu = 1/\lambda$$

THE BALANCE POINT, x_0, IS THAT POINT WHERE

$$\int_0^{x_0} f(x)\,dx = \int_{x_0}^{\infty} f(x)\,dx$$

THUS:

$$\int_0^{x_0} e^{-\lambda x}\,dx = \int_{x_0}^{\infty} e^{-\lambda x}\,dx$$

$$e^{-\lambda x}\Big|_0^{x_0} = e^{-\lambda x}\Big|_{x_0}^{\infty}$$

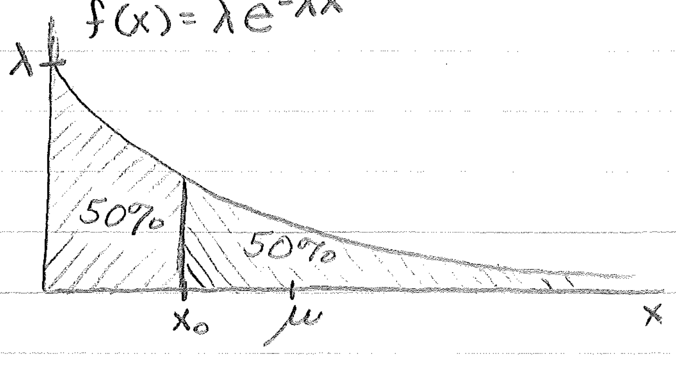
$$e^{-\lambda x_0} - 1 = -e^{-\lambda x_0}$$

OR $e^{-\lambda x_0} = \frac{1}{2}$

$$-\lambda x_0 = \ln \frac{1}{2}$$

$$\lambda x_0 = \ln 2$$

$$x_0 = \frac{1}{\lambda} \ln 2 = \mu \ln 2 < \mu$$



B. WEIBULL DISTRIBUTION

1. DENSITY:
$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^{\beta}} \mu(x)$$

TWO PARAMETERS:

$\beta \rightarrow$ SHAPE PARAMETER ($\beta \rightarrow$ BETA)

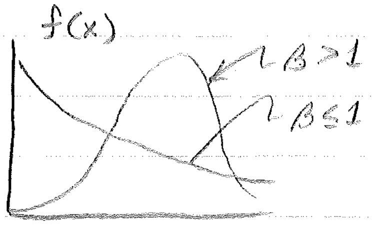
$\eta \rightarrow$ CHARACTERISTIC LIFE OR
SCALE PARAMETER ($\eta \rightarrow$ ETA)

$\Rightarrow$ FOR $\beta = 1$, $f(x) = \frac{1}{\eta} e^{-(x/\eta)} \rightarrow$ EXPONENTIAL

2. $F(x) \triangleq \int_{-\infty}^{x} f(x)dx = 1 - e^{-\left(\frac{x}{\eta}\right)^{\beta}}$

3. $R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$

4. $h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$



η MERELY SCALES

$$\mu = E[X] = \eta\,\Gamma\left(\frac{1}{\beta} + 1\right)$$
$$\sigma^2 = VAR(X) = \eta^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]$$

WHERE
$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

FOR $n \in$ INTEGERS, $\Gamma(n+1) = n! = n\Gamma(n)$

EVALUATING $\Gamma(n)$

EXAMPLE:

$$\Gamma(3.2) = \Gamma(2.2+1) = 2.2\,\Gamma(2.2)$$
$$= 2.2\,\Gamma(1+1.2) = (2.2)(1.2)\,\Gamma(1.2)$$
$$= (2.2)(1.2)(\underset{\substack{\uparrow \\ \text{FROM} \\ \text{TABLE}}}{0.918}) = 2.42$$

GENERALLY, FOR m STEPS

$$\Gamma(n) = \underbrace{(n-1)(n-2)(n-3)\ldots(n-m)}_{m \text{ TERMS}}\,\Gamma(n-m)$$

PROBLEM:

FIND THE RELIABILITY OF AN ITEM WHICH FAILS IN ACCORDANCE WITH THE WEIBULL DISTRIBUTION AT $X = \eta$ HRS:

$$R(x) = e^{-(\frac{x}{\eta})^{\beta}} = e^{-(\frac{\eta}{\eta})^{\beta}} = e^{-1}$$

PROBLEM:

A TYPE OF ELECTRON TUBE HAS FAILURE TIMES THAT ARE WEIBULL DISTRIBUTED WITH $\eta = 10^3$ HRS, $\beta = 2$. a. WHAT IS THE PROBABILITY THAT A NEW TUBE WILL OPERATE FOR 100 HRS? b. WHAT IS THE AVERAGE LIVE. c. WHAT IS THE HAZARD AT $10.^3$ HRS?

(CONT)

(CONT)

a. FIND $P[X > 100] = R(100)$

$$= e^{-\left(\frac{x}{\eta}\right)^{\beta}}$$

$$= e^{-\left(\frac{10^2}{10^3}\right)^2}$$

$$= e^{-0.01}$$

$$= 0.99005$$

b. $\mu = \eta\, \Gamma\left[\frac{1}{\beta} + 1\right]$

$$= 10^3\, \Gamma\left(3/2\right)$$

$= 886$ HRS

c. $h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta - 1}$

$$h(10^3) = \frac{2}{10^3}\left(\frac{10^3}{10^3}\right) = 2 \times 10^{-3} = .002$$

C. NORMAL DISTRIBUTION

TWO PARAMETERS: μ; σ

1. DENSITY FUNCTION

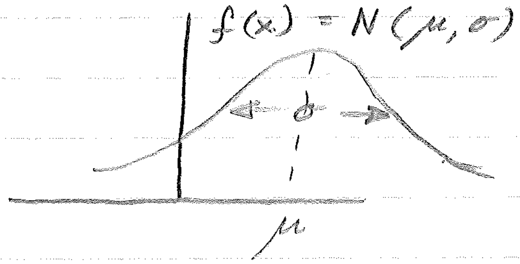
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

2. $$F(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

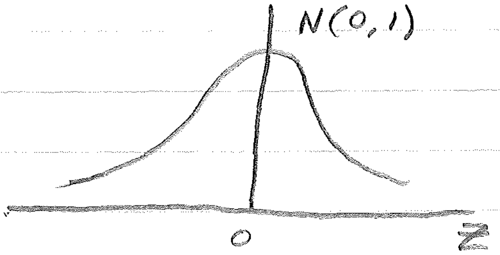
3. $R(x) = 1 - F(x)$

4. $h(x) = f(x) / R(x)$

F(x), R(x), & h(x) ARE NOT DIRECTLY COMPUTABLE, BUT ARE TABLED



A STANDARD NORMAL CURVE: N(0,1) TABLED IS $Z = \frac{x-\mu}{\sigma}$



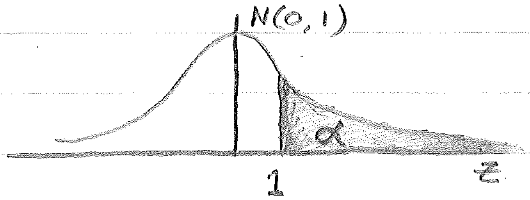
$$R(x) = P[X > x] = P\left[Z > \frac{x-\mu}{\sigma}\right] \Rightarrow f(x) = \frac{f(z)}{\sigma}$$

EXAMPLE: BRAND X TIRES HAVE WEAROUT MILEAGES THAT ARE NORMALLY DISTRIBUTED WITH $\mu = 55,000$ MILES AND $\sigma = 5000$ MILES

FIND

a. $R(60,000 \text{ MILES})$ b. $h(50,000)$

$$\text{a. } R(60,000) = P[X > 60,000] = P\left[Z > \frac{60,000-55,000}{5000}\right] = P[Z > 1]$$



FROM TABLE: $P[Z > 1] = 0.15866$

b. $h(x) = {}^{f(x)}/R(x)$

$$F(x) = P[X < 50,000] = P\left[Z < \frac{-5000}{5000}\right] = P[Z < -1] = P[Z > 1] = 0.15866$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}(5 \times 10^3)} e^{-\frac{1}{2}} = 4.839 \times 10^{-5}$$

$$R(x) = 1 - F(x) \Rightarrow R(50,000) = .84134$$

$$h(50,000) = \frac{4.839 \times 10^{-5}}{0.84134} = 5.75 \times 10^{-5}$$

CONDITIONAL

IF AN ITEM HAS OPERATED FOR TIME t_0, WHAT IS ITS RELIABILITY FOR THE SAME DEVICE FROM t_0 TO t_1 $\ni$ $t_1 > t_0$. THAT IS; FIND

$$P[T > t_1 / T > t_0]$$

LET $A: T > t_1$

$B: T > t_0$

WE THUS WISH TO FIND

$$P[A/B]$$

WE KNOW FROM PROBABILITY THAT

$$P[A \cap B] = P[A]P[B/A] = P[B]P[A/B]$$

$$\therefore \quad P[A/B] = \frac{P[A]P[B/A]}{P[B]}$$

OR EQUIVALENTLY:

$$P[T > t_1 / T > t_0] = \frac{P[T > t_1]P[T > t_0 / T > t_1]}{P[T > t_0]}$$

OBVIOUSLY

$$P[T > t_0 / T > t_1] = 1$$

THUS

$$P[T > t_1 / T > t_0] = \frac{P[T > t_1]}{P[T > t_0]} = \frac{R(t_1)}{R(t_0)}$$

D. GAMMA DISTRIBUTION

$\alpha \rightarrow$ SHAPE PARAMETER

$\lambda \rightarrow$ SCALE PARAMETER

1. DENSITY FUNCTION

$$f(x) = \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$$

IF α IS AN <u>INTEGER</u>:

2. $$F(x) = \sum_{K=\alpha}^{\infty} \frac{e^{-\lambda x}(\lambda x)^{K}}{K!} \quad \Leftarrow \text{POISSON}$$

3. $$R(x) = \sum_{K=0}^{\alpha-1} \frac{e^{-\lambda x}(\lambda x)^{K}}{K!}$$

4. $$h(x) = f(x)/R(x)$$

COMMENTS ON USE CRITERIA:

a. IF WE HAVE DATA AND CONSTANTS APPROPRIATE TO THE DISTRIBUTION

b. IF WE HAVE A CONSTANT HAZARD AND α^{TH} CAUSES SYSTEM FAILURE.

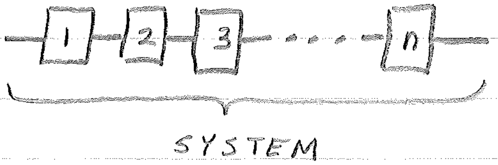
NOTE: $\alpha = 1 \Rightarrow$ EXPONENTIAL DISTRIBUTION

E. LOGNORMAL DISTRIBUTION

WILL BE DISCUSSED IN MAINTAINABILITY SECTION

SYSTEMS AND SUBSYSTEMS (SECTION III)

1. **SERIES MODEL**: COMPRISED OF n COMPONENTS (OR FUNCTIONS) WHICH MUST OPERATE FOR THE SYSTEM TO SUCCEED.



⇐ BLOCK DIAGRAM

$$P[\text{SYSTEM SUCCESS}] = P[1 \cap 2 \cap 3 \cap \ldots \cap n]$$

IF EACH COMPONENT IS INDEPENDENT,

$$P[\text{SYSTEM SUCCESS}] = P[1]P[2]P[3]\ldots P[n]$$
$$= R_S = \prod_{i=1}^{n} P(i) = \prod_{i=1}^{n} R_i$$

FOR EQUIVALENT COMPONENTS

$$R_S = R_i^{\,n}$$

EXAMPLE: AN ELECTRONIC SYSTEM CONSISTS OFF 1000 COMPONENTS WHICH ARE NECESSARY TO SYSTEM SUCCESS. EACH COMPONENT HAS $R(37) = 0.999$. WHAT IS $R_S(37)$?

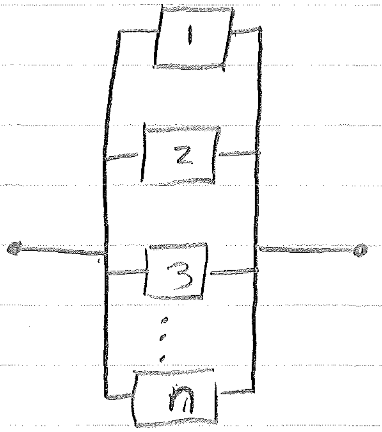
$$R_S(37) = [R(37)]^{1000}$$
$$= [0.999]^{1000}$$
$$= 0.3676954$$

TO INCREASE RELIABILITY,

1. APPLY REDUNDANCY
2. REDUCE COMPLEXITY → DESIGN
3. INCREASE RELIABILITY OF COMPONENTS

2. REDUNDANCY

a. SIMPLE ACTIVE REDUNDANCY - ONLY ONE COMPONENT IS NEEDED FOR SUCCESS. ALL COMPONENTS ARE OPERATING



$$R_s = P[1 \cup 2 \cup 3 \cup \ldots \cup n]$$

$$F_s = 1 - R_s = Q_s = P[\bar{1} \cap \bar{2} \cap \bar{3} \cap \ldots \cap \bar{n}]$$

FOR INDEPENDENT COMP

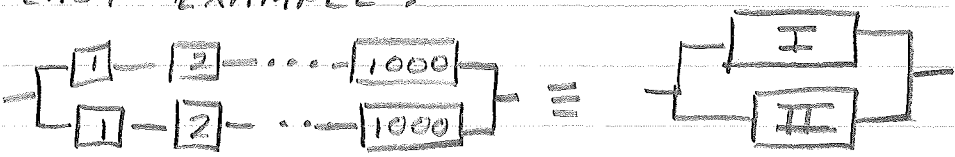
$$Q_s = P[\bar{1}]P[\bar{2}]P[\bar{3}]\cdots P[\bar{n}]$$

$$= Q_1 \, Q_2 Q_3 \ldots Q_n$$

$$= \prod_{i=1}^{n} Q_i$$

FOR IDENTICAL COMPONENTS: $Q_s = Q_i^n$

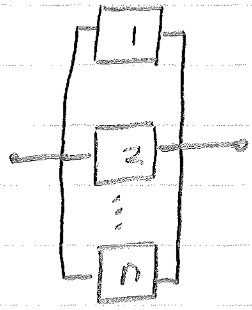
EXAMPLE: CONSIDER, A REDUNDANT CASE OF THE LAST EXAMPLE:



$$Q_s = Q_I Q_{II} = (1 - 0.3676954)^2 = 0.399809$$

$$\Rightarrow R_s = 1 - Q_s = 0.600191$$

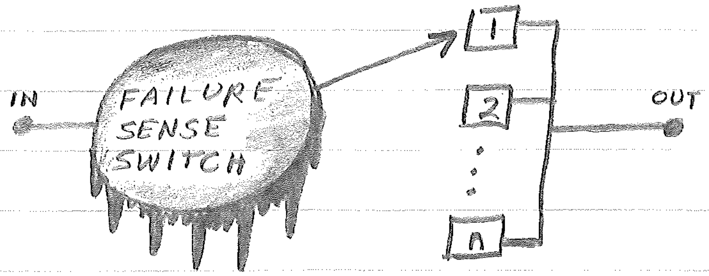
b. PARTIAL ACTIVE REDUNDANCY: AT LEAST SOME MINIMUM NUMBER K MUST SUCCEED.



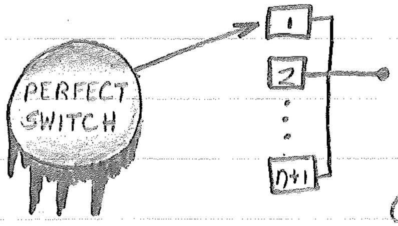
ASSUMING IDENTICAL INDEPENDENT COMPONENTS WE THUS EMPLOY THE BINOMIAL DISTRIBUTION

$$R_s = P[X \geq k]$$
$$= \sum_{x=k}^{n} \binom{n}{x} R^x Q^{n-x}$$

c. STANDBY REDUNDANCY



i. A SPECIFIC TIME DEPENDENT MODEL



IDENTICAL ELEMENTS WITH EXPONENTIAL DISTRIBUTION

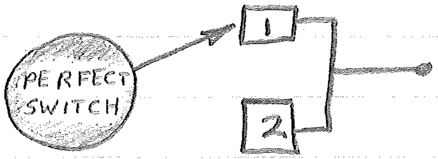
(ie $\lambda_1 = \lambda_2 = \ldots = \lambda_n = \lambda_{n+1}$)

APPLY POISSON DISTRIBUTION:

$$R_S = P[\underline{K} \leq n] \ni n = \# \text{ FAILURES}$$

$$= \sum_{K=0}^{n} \frac{(\lambda x)^K e^{-\lambda x}}{K!}$$

ii. ANOTHER SPECIFIC TIME DEPENDENT CASE

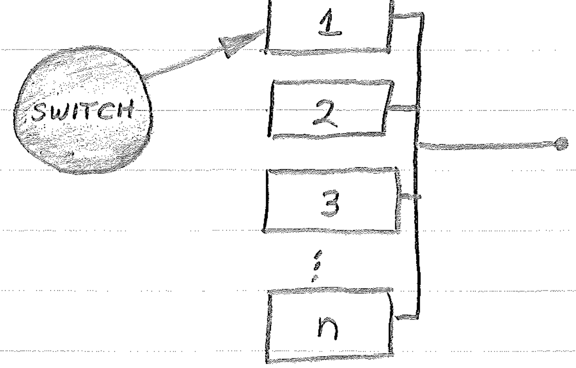


EXPONENTIALLY WITH $\lambda_1 \neq \lambda_2$

$$R_S = \frac{\lambda_2 e^{-\lambda_1 x} - \lambda_1 e^{-\lambda_2 x}}{\lambda_2 - \lambda_1}$$

iii. A N NON-TIME DEPENDENT MODEL

(ie ELEMENT FAILURE NOT A FUNCTION OF TIME)



ASSUME NO UPWARD SWITCHING (ie NO GO FROM 2 TO 1)

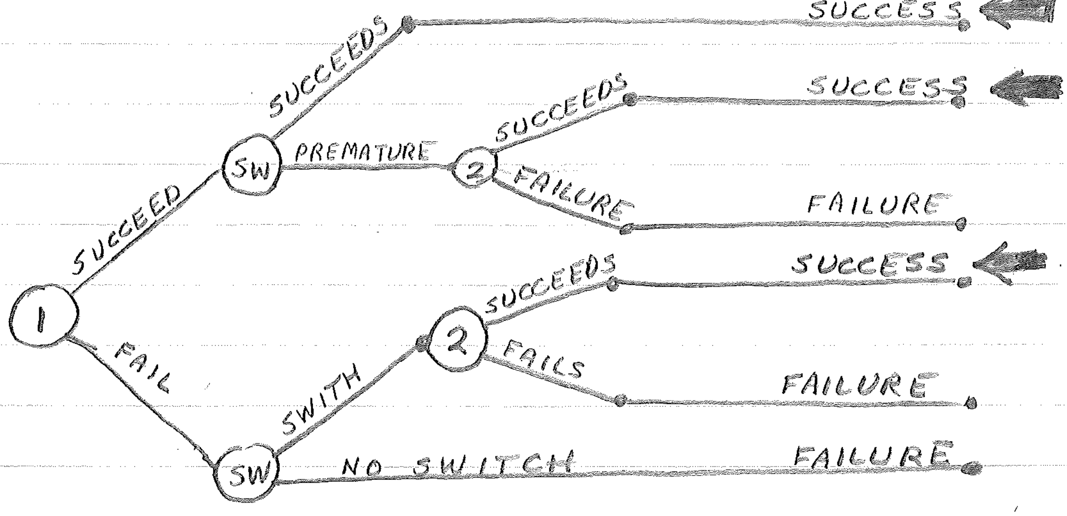
$p_i = P[i^{TH}$ ELEMENT WORKS$]$

$q_i = 1 - p_i$

$q_W = P[$FAILURE TO SWITCH / IT IS APPROPRIATE$]$

$q_W' = P[$PREMATURE SWITCHING$]$

TREE DIAGRAM FOR $n = 2$



THE SUCCESSES ARE MUTUALLY EXCLUSIVE

$$\Rightarrow R_S = P_1 P_\omega' + P_1 q_\omega' P_2 + q_1 P_\omega P_2$$

3. APPORTIONMENT

R_s^* = DESIRED RELIABILITY OF THE SYSTEM

GENERALLY:

$$R_s = f[R_i^*] = f[R_1, R_2^*, \ldots \quad]$$

A CASE:

1. SERIES, IDENTICAL, INDEPENDENT

$$R_s^* = R_i^n \Rightarrow R_i = \sqrt[n]{R_s^*}$$

2. PARALLELL

$$Q_i = \sqrt[n]{Q_s^*}$$

AGREE METHOD

ASSUMES: A "SERIES" OF INDEPENDENT, EXPONENTIALLY FAILING SUBSYSTEMS

CONSIDERS: 1. DIFFERENT SUBSYSTEM MISSION TIME
2. IMPORTANCE OF SUBSYSTEMS
3. SUBSYSTEM COMPLEXITY

ACCEPTABLE MEAN LIFE OF i^{TH} SUBSYSTEM:

$$\theta_i \simeq \frac{N w_i t_i}{n_i [-\ln R^*(t)]}$$

$$\Rightarrow R_i^*(t_i) \simeq e^{-t_i/\theta_i}$$

(Pp. III-23-5)

ARINC METHOD (BRUTE FORCE)

ASSUMES: INDEPENDENT SUBSYSTEM

: EXPONENTIALLY DISTRIBUTED FAILURES

CONSIDERS; CURRENT SYSTEM STATUS

ACCROSS THE BOARD IMPROVEMENTS MUST BE MADE IN ALL SUBSYSTEMS;

$$\lambda_S = \sum_{i=1}^{n} \lambda_i \quad ; \quad R_{SYS}(t) = e^{-\lambda_S t}$$

$$= e^{-\sum \lambda_i t}$$

(SEE pp. 25, 26, SEC III)

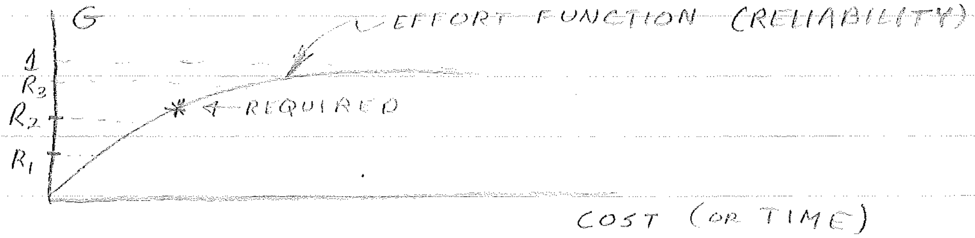
MINIMIZATION OF EFFORT

ASSUMES: ELEMENTS ARE IN SERIES

: ELEMENTS FAIL INDEPENDENTLY

: EQUAL EFFORT FUNCTIONS

CONSIDERS: CURRENT SYSTEM STATUS



(CONT.)

FOR K_0, MAXIMIZE j ∋

$$R_j < \left[\frac{R^*}{\prod_{i=j+1}^{n+1} R_i} \right]^{\frac{1}{j}} = r_j$$

$$R_1 < R_2 \leq R_3 \ldots \leq R_n \leq R_{n+1} = 1$$

EXAMPLE; (NOTE ORDERED RELIABILITIES)

$R_1 = 0.8$; $R_2 = 0.85$, $R_3 = 0.90$ $[R_4 = 1.0]$

$R^*_{SYS} = 0.70$

1. LET $j = 1$

IS $R_1 < \left[\frac{0.7}{(0.85)(0.9)1}\right]^{1/1} = 0.915 = r_1$

YES

∴ WE NEED ONLY RAISE R_1 TO 0.915.

(BUT IS IT THE MOST EFFORTLESS?)

2. LET $j = 2$

IS $R_2 < \left[\frac{0.7}{(0.9)1}\right]^{1/2} = 0.882 = r_2$

YES

∴ WE CAN RAISE R_1 & R_2 TO 0.882

(BUT IS IT THE CHEAPEST?)

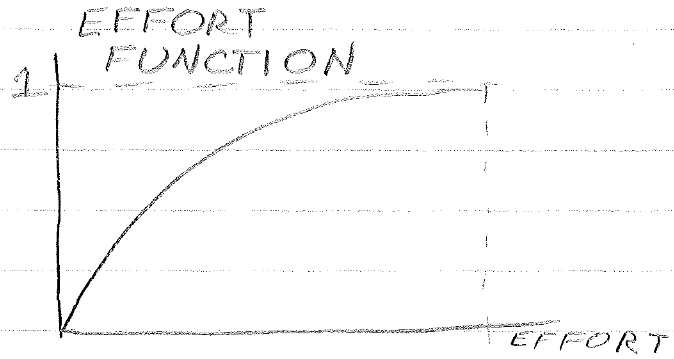
3. LET $j = 3$

IS $R_3 < \left[\frac{0.7}{1}\right]^{1/3} = 0.888 = r_3$ NO

∴ WE CAN RAISE ALL THREE TO 0.888

⇒ $j = 2$ CASE IS OPTIMMAL

LET: $R_1^* = R_2^* = 0.882$



4. MONTE CARLO SIMULATION ~

SWITCHING WITH IMPERFECT SWITCHES

⇒ NON TIME DEPENDENT FAILURES

MONTE CARLO EXAMPLE:

SIMULATE COIN FLIPPING

TRIAL	#	HEAD 0-4	TAIL 0-9
1	9		✓
2	2	✓	
3	2	✓	
4	7		✓
5	6		✓
6	2	✓	
7	2	✓	
8	4	✓	
9	5		✓
10	2	✓	
		=6	=4

THUS, THRU EXPERIMENT:

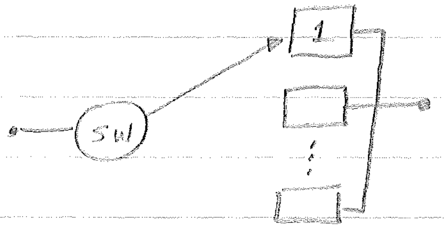
$$P[HEAD] = \frac{6}{10}$$

FIND P[HEAD & TAIL ON TWO TOSSES]

TRIAL	#'S	0-4 HEAD	5-9 TAIL	HH	HT	TT
1	34	XX		X		
2	63	X	X		X	
3	22	XX		X		
4	69		XX			X
5	11	XX		X		
6	57		XX			X
7	46	X	X		X	
8	19	X	X		X	
9	41	XX		X		
10	65		XX			X

$P[H \cap H] = \frac{4}{10}$; $P[T \cap T) = \frac{3}{10}$

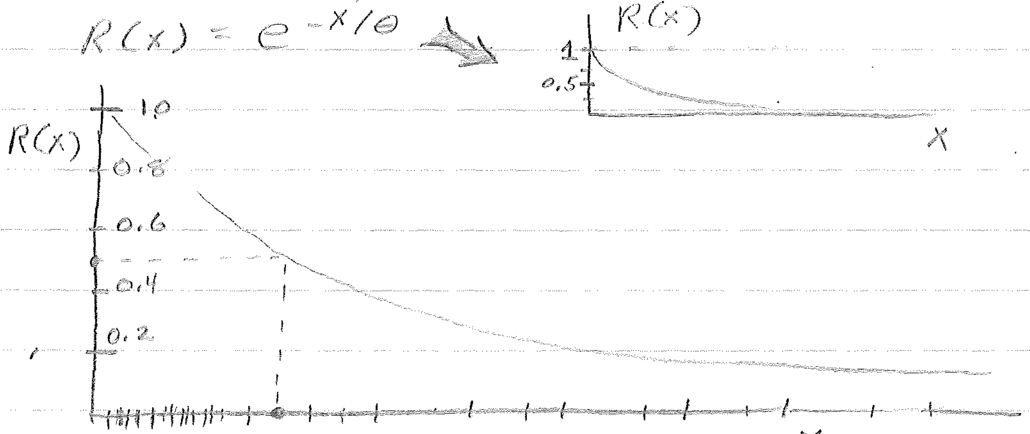
$P[H \cap T] = \frac{3}{10}$



ASSUME ELEMENT 1 FAILS EXPONENTIALLY

$\theta = 100$ HRS ; $X = 50$ HRS

$R(x) = e^{-x/\theta}$



PICKING RANDOM PROBABILITIES AND FINDING CORRESPONDING TIMES. VALUES WILL TEND TO BUNCH UP AROUND ORIGIN

CONSIDER ;

1. $F(x) = (RN) = 1 - e^{-x/\theta}$; RN = RANDOM #)

   $\Rightarrow x = -\theta \ln(1 - RN)$

2. $R(x) = RN$

   $\Rightarrow x = -\theta \ln RN$

USING $R(x) = e^{-\frac{x}{\theta}}$; $\theta = 100$ HRS; $X = 50$ HRS

TRIAL	RN	$\ln(RN)$	$x = -\theta \ln RN$	SUCCESS ?
1	0.15	-1.897	189.7 HR	YES
2	0.89	-0.163	16.3 HR	NO
3	0.49	-0.713	71.3 HR	YES

RELIABILITY DEMONSTRATION & TESTING

CONCERNED PRIMARILY WITH

1. PARAMETER ESTIMATION $(\beta\ \eta, \sigma, \mu, \theta, \lambda)$ (STRICTLY CONCERNED WITH TEST INFORMATION)
2. HYPOTHESIS TESTING AND/OR ACCEPTANCE LIFE TESTING (CONCERNED WITH DECISION MAKING, ie, IS PARAMETER ESTIMATION GOOD OR BAD?)

TYPES OF PARAMETER ESTIMATION

1. POINT ESTIMATION ~ SINGLE BEST GUESS

   EX: 5 ITEMS ARE PLACED ON LIFE TEST WITH THE RESULTS THAT 4 SURVIVE. THE BEST ESTIMATE FOR RELIABILITY, $\hat{R}$

   IS: $\hat{R} = \dfrac{\text{SUCCESSES}}{\#\text{OF TRIALS}} = \dfrac{d}{n}$

   FOR CASE @ HAND, $\hat{R} = \frac{4}{5} = 80\%$

   $\hat{R} = \dfrac{d}{n} \Rightarrow$ BINOMIAL APPROACH (A NONPARAMETRIC TEST)

2. CONFIDENCE INTERVALS

A. BINOMIAL APPROACH

WHAT R(x) WILL YIELD AN α CHANCE OF GETTING OVER d OBSERVED SUCCESSES?

TO FIND OUT, WE USE THE BINOMIAL:
(BERNOULLI TRIAL: A GO, NO/GO TEST)
HAVE n BERNOULLI TRIALS. FIND

$$P[X \geq x] = \alpha$$

$$= \sum_{k=d}^{n} \binom{n}{k} p^k q^{n-k}$$

EXAMPLE: 3 ITEMS TESTED SUCH THAT 2 SURVIVE.

$d = 2, r = 1, n = 3, X = 37$ HRS

$\hat{R}(37) = \frac{2}{3} = 0.66666666666667$

LET $\alpha = 0.05$ $(= 5\%)$

$\therefore$ WE WANT

$$0.05 = \sum_{k=2}^{3} \binom{3}{k} p^k q^{n-k}$$

$$= 3p^2 q + p^3$$

$$= 3p^2(1-p) + p^3$$

$$= 3p^2 - 2p^3$$

$$\Rightarrow p = 0.135 = R(37)$$

$\therefore$ WE ARE 95% $(1-\alpha)$ CONFIDENT THAT THE TRUE RELIABILITY OF THIS ITEM IS $R(37) \geq 0.135$)

HOW HI CAN $R(x)$ BE AND STILL GIVE US THE RESULTS WE GOT OR WORSE! i.e. WHAT $R(37)$ WILL GIVE US A α CHANCE OF GETTING $< d$ SUCCESSES

$$\alpha = \sum_{k=0}^{2} \binom{3}{k} p^d q^{n-d}$$

$$= 0.05 = q^3 + 3pq^2 + 3p^2q$$

$$\Rightarrow R(37) \cong 0.983$$

OR

WE ARE 95% CONFIDENT THAT $R(37) \leq 0.983$

COMBINIG WITH PREVIOUS SOLUTION:
WE ARE 90% CONFIDENT THAT
$0.135 \leq R(37) \leq 0.983$

90% CONF. INTERVAL

SUCCESS	TRIALS	
2	3	$0.153 \leq R(37) \leq 0.983$
4	6	$0.271 \leq R(37) \leq 0.937$
20	30	$0.501 \leq R(37) \leq 0.807$

ORDER STATISTICS

ONE MAY ALSO EMPLOY THE TIMES AT WHICH THE ITEMS HAVE FAILED FOR MORE INFO.

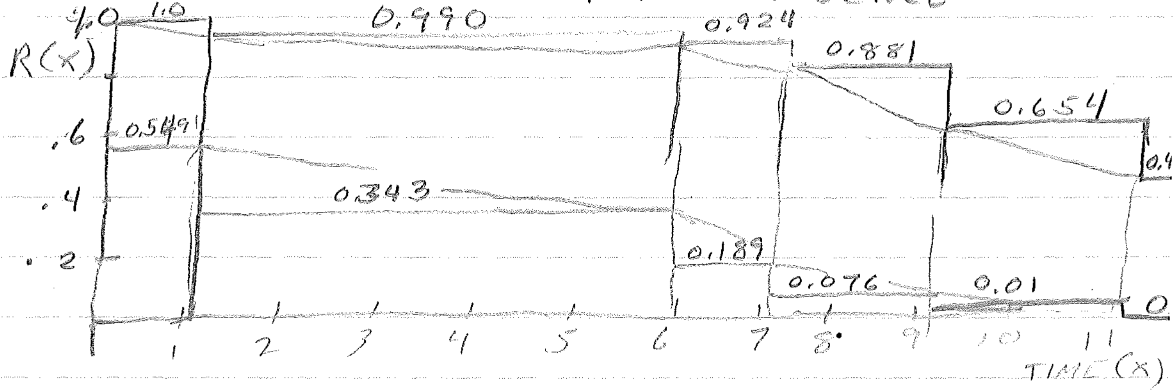
ORDER STATISTICS - NON PARAMETRIC TEST WHICH CONSIDERS ITEM FAILURE TIMES

EXAMPLE (P_g IV-13)

i	X_i
1	1
2	6
3	7
4	9
5	11

ORDERED

90% CONFIDENCE



WHEN A FAILURE IS ENCOUNTERED, APPLY THE BINOMIL PRIOR LOWER LIMIT & FOLLOWING UPPER LIMIT.

3. PARAMETRIC TECHNIQUES

(GENERALLY MORE PERCISE)

a. GRAPHICAL (FOR EXPONENTIAL, WEIBULL, & NORMAL)

i. EXPONENTIAL GRAPHICAL PROCEDURE

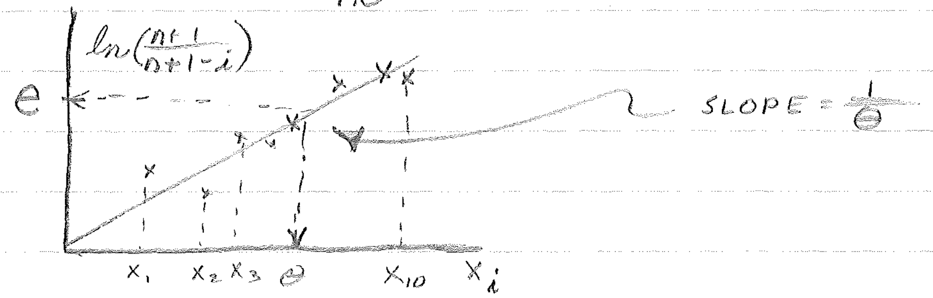
θ IS PARAMETER OF INTEREST

- n ITEMS TESTED UNTIL ALL FAIL
- FAILURE TIMES ORDERED: $x_1, x_2, \ldots, x_n$

$\ni x_1 \leq x_2 \leq \ldots \leq x_n$

$R(x) = e^{-x/\theta}$

i	x_i	$\frac{n+1}{n+1-i} \leftarrow$ ESTIMATE OF $1/R(x)$
1	x_1	$11/10 = 1.1$
2	x_2	$11/9 = 1.2\overline{2}$
3	x_3	$11/8 =$
4	x_4	$11/7$
⋮		⋮
10		$= 1.0$



$\ln R(x) = -\frac{1}{\theta} x \leftarrow$ ARITHMETIC SLOPE

ii. WEIBULL GRAPHING PROCEDURE

FOR WEIBULL:

$$R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$$

$$\ln R(x) = -\left(\frac{x}{\eta}\right)^{\beta}$$

$$\ln \frac{1}{R(x)} = \left(\frac{x}{\eta}\right)^{\beta}$$

$$\ln\left[\ln \frac{1}{R(x)}\right] = \beta \ln\left(\frac{x}{\eta}\right)$$

$$= \beta\left[\ln x - \ln \eta\right]$$

NOW

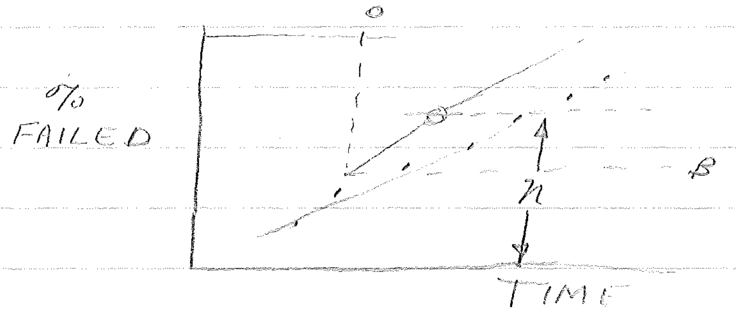
$$\ln\left[\ln \frac{1}{R(x)}\right] = \ln\left[\ln \frac{1}{1-F(x)}\right]$$

∴ Y AXIS INTERCEPT = $\beta \ln \eta$

SLOPE = β

ORDERED DATA

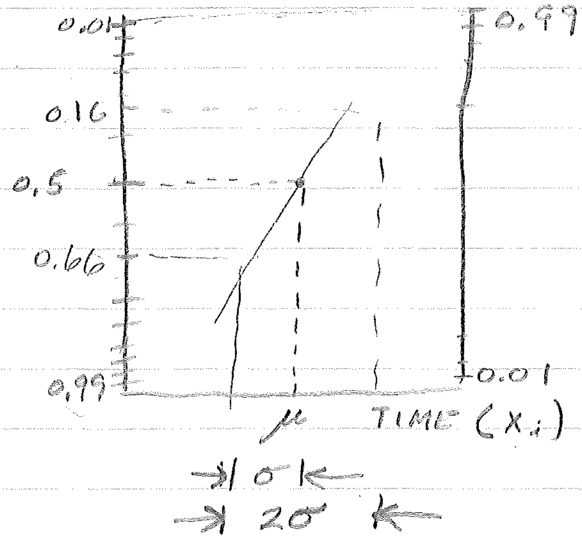
i	x_i	$\frac{100i}{n+1}$ = % FAILED
1	x_1	$\frac{100}{n+1}$
2	x_2	$\frac{200}{n+1}$
⋮	⋮	⋮
n	x_n	$\frac{100n}{n+1}$



iii. NORMAL GRAPHICAL PROCEDURES

n ITEMS TESTED, ALL FAIL

i	X_i	$\frac{100i}{n+1} \simeq F(x)$
1	X_1	
2	X_2	
$\vdots$	$\vdots$	
n	X_n	



iv. HAZARD PLOTTING (ALL ITEMS DON'T FAIL)
r FAILURES OUT OF n.
SUSPENDED ITEMS ← REMOVED FOR TESTING
IN "HAZARD PLOTTING FOR INCOMPLETE
FAILURE DATA" - JOURNAL OF QUALITY
TECHNOLOGY VOL I, #1, JAN 1969.

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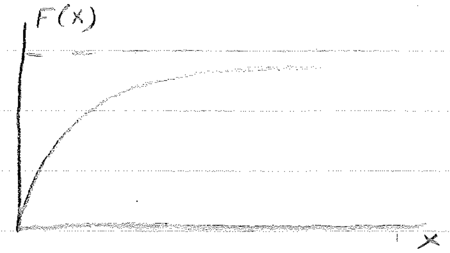
b. GOODNESS OF FIT TEST

χ^2 TEST	KOLMOGOROV-SMIRNOV
DISCRETE OR CONTINUOUS	ONLY FOR CONTINUOUS DISTRIBUTION
ALLOWS US TO ESTIMATE PARAMETERS	CANNOT MAKE PARAMETER(S) ESTIMATES (ie: PARAM. PART OF TEST)
REQUIRES LARGE SAMPLE SIZE: $n \geq 30$	CAN EMPLOY SMALL SAMPLE SIZES

i. χ^2 TEST:



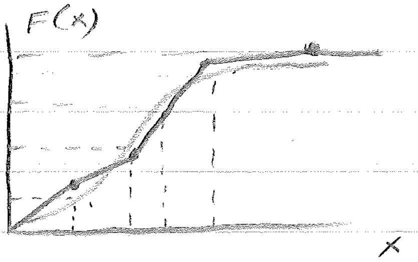
1. ASSUME DISTRIBUTION
2. PICK $\overset{\text{OPTIMALLY, MAKE } \alpha \text{ BIG}}{\alpha} = P[\text{REJECTION} / \text{TRUE DISTRIBUTIONS}]$
3. GROUP DATA INTO i INTERVALS
4. DETERMINE SAMPLES IN EACH INTERVAL $= O_i$
5. DETERMINE $E[\text{SAMPLES IN INTERVAL}] = E_i$
6. $$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$
7. IF $$\chi^2 > \chi^2_{\alpha;\, k-p-1}$$

→ TOTAL # OF INTERVALS
→ # OF PARAMETERS IN ASSUMED DISTRIBUTION

REJECT HYPOTHESIS

$x = e^{-x} = \ln \frac{1}{x} \Rightarrow x = 0.5671432904$

iii. KOLMOGOROV-SMIRNOV TEST (K-S TEST)



1. DETERMINE UNDERLYING DISTRIBUTION
2. DETERMINE LEVEL OF SIGNIFICANCE, α
3. COMPUTE, FROM DISTRIBUTION OF PARAMETERS,

   $F(x_i) = P[X < x_i] \ni x_i = i^{TH}$ SAMPLE
4. COMPUTE, FROM DATA, $\hat{F}(x_i)$ = PART OF SAMPLE OBSERVATIONS $\leq x_i$
5. FIND MAX VALUE OF $|F(x_i) - \hat{F}(x_i)| = d$
6. REJECT DISTRIBUTION IF $d > d_{\alpha,n}$

4. PARAMETER ESTIMATION

a. EXPONENTIAL

- TEST TYPES

	WITH REPLACEMENT	W/O REPLACE
TIME TERMINATED	✓	✓
FAILURE TERMINATED	✓	UNTIL ALL FAIL (COMPLETE TEST)

POINT ESTIMATE: $\hat{\theta} = \frac{\text{TOTAL ITEM HOURS}}{\text{TOTAL \# OF FAILURES}} = \frac{1}{r}\sum X_i$

LET: $\hat{\theta} = MTBF = \frac{1}{r} X_t$

WHERE $X_t = \sum_{i=1}^{n} X_i$ & $r = $ # FAILURES

i. FAILURE TERMINATED TEST w/o REPLACE.

$n = $ ITEMS ON TEST

$r = $ FAILURES

THEN: $X_t = (n-r) X_r + \sum_{i=1}^{r} X_i$

$$\Rightarrow \hat{\theta} = \frac{X_t}{r} = \frac{(n-r)X_R + \sum_{i=1}^{r} X_i}{r}$$

ii. FAILURE TERMINATED WITH REPLACE.

$X_t = n X_r \ni X_r = $ TIME OF r^{TH} FAILURE

$$\Rightarrow \hat{\theta} = \frac{n}{r} X_t$$

FOR FAILURE TERMIN., A $(1-\alpha)$ CONFIDENCE INTERVAL IS

$$\frac{2r\hat{\theta}}{\chi^2_{\frac{\alpha}{2};2r}} \leq \theta \leq \frac{2r\hat{\theta}}{\chi^2_{1-\frac{\alpha}{2};2r}}$$

OR, SINCE $\theta = 1/\lambda$

$$\frac{\chi^2_{\alpha/2;2r}}{2r\hat{\theta}} \geq \lambda \geq \frac{\chi^2_{1-\alpha/2;2r}}{2r\hat{\theta}}$$

NOW: $\hat{R}(x) = e^{-x/\hat{\theta}}$

THUS, FOR FAILURE TERMINATED TESTING

$$e^{-x\left[\frac{\chi^2_{\alpha/2,2r}}{2r\hat{\theta}}\right]} \leq R(x) \leq e^{-x\left[\frac{\chi^2_{1-\frac{\alpha}{2};2r}}{2r\hat{\theta}}\right]}$$

IS A $(1-\alpha)$ CONFIDENCE INTERVAL FOR $R(x)$

iii) TIME TERMINATED w/o REPLACEMENT (@ $x = T$)

$$X_t = \sum_{i=1}^{r} X_i + (n-r)T \Rightarrow \hat{\Theta} = \frac{\sum_{i=1}^{r} X_i + (n-r)T}{r}$$

iv) TIME TERMINATED WITH REPLACEMENT

$$X_t = nT \Rightarrow \hat{\theta} = nT/r$$

FOR <u>TIME TERMINATED</u>, A $(1-\alpha)$ CONF. INTERVAL IS

$$\frac{2X_t}{\chi^2_{\frac{\alpha}{2};2r+2}} \leq \Theta \leq \frac{2X_t}{\chi^2_{1-\frac{\alpha}{2};2r}}$$

EXAMPLE: TIME TERMINATED TEST, EXPONENTIAL

$n = 1$, $r = 1$, $X_t = 48{,}000$

$\hat{\theta} = 48{,}000 = \frac{X_t}{r}$

WHAT IS 90% CONF. INTERVAL?

$$\frac{2X_t}{\chi^2_{\frac{\alpha}{2};2(r+1)}} \leq \Theta \leq \frac{2X_t}{\chi^2_{1-\frac{\alpha}{2};2r}}.$$

$\alpha = .10$, $r = 1$

$$\frac{2(48\times10^3)}{\chi^2_{0.05;3} = 9.488} \leq \Theta \leq \frac{96\times10^3}{\chi^2_{0.95;2} = 0.103}$$

10,118 MILES $\leq \Theta \leq$ 932,039

b. NORMAL

TEST n ITEMS TILL THEY ALL FAIL (ie NO REPLACEMENT)

POINT ESTIMATES: $\bar{X} = \Sigma X_i / n = \hat{\mu}$

$$S^2 = \frac{\Sigma (x - \bar{x})^2}{n-1} = \hat{\sigma}^2$$

CONFIDENCE INTERVALS:

$$\bar{x} - t_{\alpha/2; n-1} \left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{\alpha/2, n-1} \left(\frac{s}{\sqrt{n}}\right)$$

$$\frac{(n-1) s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} \le \sigma^2 \le \frac{(n-1) s^2}{\chi^2_{1-\frac{\alpha}{2}; n-1}}$$

APPROXIMATELY, A $1-\alpha$ CONF. INTERVAL FOR R IS

$$\hat{R}(x) \pm Z_{\alpha/2} \sqrt{V(\hat{R}(x))}$$

WHERE

$$V[\hat{R}(x)] = \frac{[f(z = \frac{x - \bar{x}}{s})^2][1 + \frac{1}{2}(\frac{x - \bar{x}}{s})^2}{n}$$

C. WEIBULL PARAMETER ESTIMATION

i. MATCHING MOMENTS

n ITEMS TESTED UNTIL ALL FAIL

COMPUTE $\bar{x}$ & s

THEN

$$1 + \frac{s^2}{\bar{x}^2} = \frac{2\,\Gamma(2b)}{b\,\Gamma^2(b)} \qquad ; \; b = \frac{1}{\beta}$$

$$b = \frac{1}{\beta}$$

$$\eta = \frac{\bar{x}}{\Gamma(1+b)}$$

SO SOLVE, SET UP TABLE

b	$2\,\Gamma(2b)$	$b\,\Gamma^2(b)$	$\frac{2\Gamma(2b)}{b\Gamma^2(b)} \overset{?}{=} 1+\frac{s^2}{\bar{x}^2}$

SOLVING GIVES b WHICH GIVES β & η

→ BUT ! ! ! !

b IS TABLED AS FUNCTIONS OF $\left(\frac{\bar{x}}{s}\right)^2$

ii. MAXIMUM LIKELIHOOD ITERATION (n ITEMS, ALL FAIL)

1. ASSUME TRIAL VALUE FOR $\beta = \hat{\beta}$
2. COMPUTE: $\hat{\eta} = \exp\left[\frac{1}{\hat{\beta}} \ln \frac{1}{n} \sum_{i=1}^{n} x_i^{\hat{\beta}}\right]$
3. COMPUTE: $\hat{\beta}_1 = \dfrac{n}{\sum_{i=1}^{n} \left(\frac{x_i}{\hat{\eta}}\right)^{\hat{\beta}} \ln x_i - \sum_{i=1}^{n} \ln x_i}$
4. IF $\hat{\beta}_1 = \hat{\beta}$, THEN THE SOLUTION IS FOUND
   IF NOT, REPEAT STEPS 2 & 3
   WITH $\hat{\beta}_{NEW} = \frac{1}{3}\left[\hat{\beta} + 2\hat{\beta}_1\right]$

iii. PT. ESTIMATE OF RELIABILITY:

$$\hat{R}(x) = e^{-\left(\frac{x}{\hat{\eta}}\right)^{\hat{\beta}}}$$

iv. FOR LARGE SAMPLE SIZE, CONFIDENCE INTERVAL FOR $R(x)$ IS

$$\hat{R}(x) \pm Z_{\alpha/2}\sqrt{V[\hat{R}(x)]}$$

WHERE $V[\hat{R}(x)]$ IS GIVEN ON Pg IV-69 AND DEVELOPED PRIOR

v. <u>IF β IS KNOWN</u>, THE QUANTITY X^{β} IS EXPONENTIALLY DISTRIBUTED WITH $\theta = \eta^{\beta}$

THEN:

$$\hat{R}(x) = \exp\left[\frac{-x^{\beta} r}{\sum_{i=1}^{r} x_i^{\beta} + (n-r)x_r^{\beta}}\right]$$

$$\exp\left[\frac{-x^{\beta}\chi^2_{\alpha/2;2r}}{2\left[\sum_{i=1}^{r} x_i^{\beta} + (n-r)x_r^{\beta}\right]}\right] \le R(x) \le \exp\left[\frac{-x^{\beta}\chi^2_{1-\alpha/2,2r}}{2\left[\sum_{i=1}^{r} x_i^{\beta} + (n-r)x_r^{\beta}\right]}\right]$$

NOTE THIS IS FAILURE TERMINATED EXPERIMENT

ii. MAXIMUM LIKELIHOOD ITERATION (n ITEMS, ALL FAIL)

1. ASSUME TRIAL VALUE FOR $\beta = \hat{\beta}$

2. COMPUTE: $\hat{\eta} = \exp\left[\frac{1}{\hat{\beta}} \ln \frac{1}{n} \sum_{i=1}^{n} x_i^{\hat{\beta}}\right]$

3. COMPUTE: $\hat{\beta}_1 = \dfrac{n}{\sum_{i=1}^{n} \left(\frac{x_i}{\hat{\eta}}\right)^{\hat{\beta}} \ln x_i - \sum_{i=1}^{n} \ln x_i}$

4. IF $\hat{\beta}_1 = \hat{\beta}$, THEN THE SOLUTION IS FOUND
IF NOT, REPEAT STEPS 2 & 3
WITH $\hat{\beta}_{NEW} = \frac{1}{3}\left[\hat{\beta} + 2\hat{\beta}_1\right]$

iii. PT. ESTIMATE OF RELIABILITY:

$$\hat{R}(x) = e^{-\left(\frac{x}{\hat{\eta}}\right)^{\hat{\beta}}}$$

iv. FOR LARGE SAMPLE SIZE, CONFIDENCE INTERVAL FOR $R(x)$ IS

$$\hat{R}(x) \pm Z_{\alpha/2} \sqrt{V[\hat{R}(x)]}$$

WHERE $V[\hat{R}(x)]$ IS GIVEN ON Pg IV-69 AND DEVELOPED PRIOR

v. IF β IS KNOWN, THE QUANTITY X^{β} IS EXPONENTIALLY DISTRIBUTED WITH $\theta = \eta^{\beta}$

THEN:

$$\hat{R}(x) = \exp\left[\frac{-x^{\beta} r}{\sum_{i=1}^{r} X_i^{\beta} + (n-r) X_r^{\beta}}\right].$$

$$\exp\left[\frac{-x^{\beta} \chi^2_{\alpha/2;2r}}{2\left[\sum_{i=1}^{r} X_i^{\beta} + (n-r) X_r^{\beta}\right]}\right] \leq R(x) \leq \exp\left[\frac{-x^{\beta} \chi^2_{1-\alpha/2,2r}}{2\left[\sum_{i=1}^{r} X_i^{\beta} + (n-r) X_r^{\beta}\right]}\right].$$

NOTE THIS IS FAILURE TERMINATED EXPERIMENT

5. TEST OF HYPOTHESIS

P[REJECT HYPOTHESIS] = 1 - P[ACCEPTING HYPOTHESIS]

= CONFIDENCE FOR SINGLE VALUE REQUIREMENT

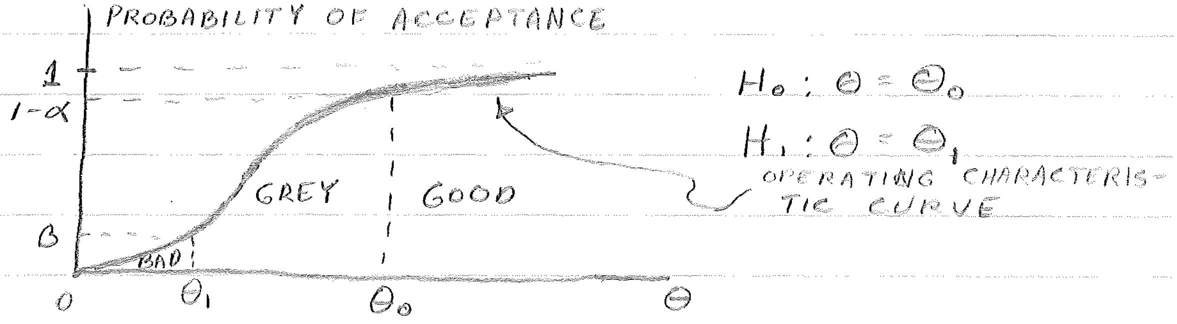
H_0: NULL HYPOTHESIS [EX: $\Theta = \Theta_0$] "GOAL"

H_1: ALTERNATE HYPOTHESIS [EX: $\Theta < \Theta_0$] "

α RISK = P[REJECTING H_0 / H_0 IS CORRECT]

β RISK = P[ACCEPTING H_0 / H_0 IS INCORRECT]

GRAPHICAL ILLUSTRATION:



α RISK ⇔ PRODUCER'S RISK

β RISK ⇔ CONSUMER'S RISK

- IF THE HYPOTHESIS TEST IS PASSED (ie H_0 NOT REJECTED) WE WILL HAVE $(1-\beta)$ CONFIDENCE THAT $\Theta \geq \Theta_1$
- IF THE TEST HAS FAILED (ie H_0 REJECTED IN FAVOR OF H_1) WE WILL HAVE $(1-\alpha)$ CONFIDENCE THAT $\Theta \leq \Theta_0$

a. EXPONENTIAL

θ = POPULATION MEAN LIFE

$\hat{\theta}$ = POINT ESTIMATE OF θ

θ_0 = GOOD (DESIRED, GOAL) VALUE OF MEAN LIFE

θ_1 = BAD (MINIMUM ACCEPTABLE) VALUE OF MEAN LIFE

i. FOR FAILURE TERMINATED TEST

$H_0 : \theta = \theta_0 \Rightarrow \alpha$ RISK

$H_1 : \theta < \theta_0$

– COMPUTE TEST STATISTIC : $\hat{\theta}$

– ACCEPTANCE REGION (ACCEPT H_0) IF

$$\hat{\theta} \geq \frac{\theta_0 \chi^2_{1-\alpha;2r}}{2r}$$

EX: TEST FORMAT (pg III-88)

$n = 5$; $r = 3$; $\alpha = 0.05$

$H_0 : \theta = 100$

$H_1 : \theta < 100$

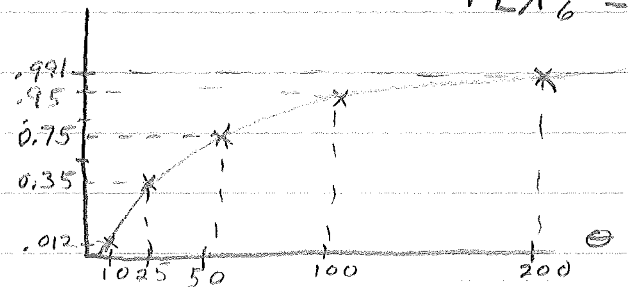
TEST STATISTIC = $\hat{\theta} = 27.3$

ACCEPT H_0 IF

$$\hat{\theta} \geq \frac{100 \; \chi_{0.95;6}}{6} \doteq 27.3$$

O.C CURVE FROM: $P[\text{ACCEPT}] = P\left[\chi^2_{2r} \geq \frac{\theta_0}{\theta} \chi^2_{1-\alpha;2r}\right.$

$= P\left[\chi^2_6 \geq \frac{164}{\theta}\right]$



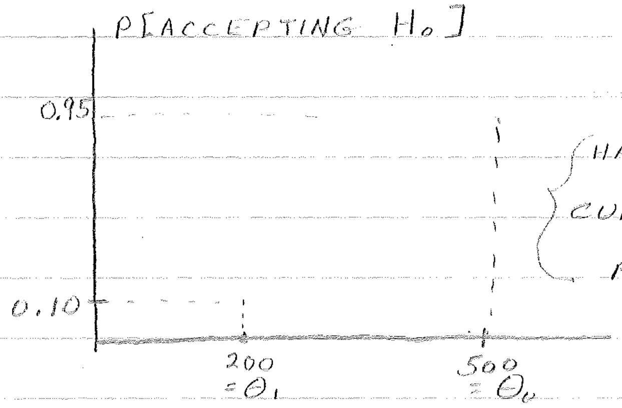
FROM TABLE D (χ^2) USING LINEAR INTERPOLATI

FROM TEST: X_i : 20, 40, 50 $\Rightarrow \bar{X} = \hat{\theta} = 70$ HRS

ACCEPT H_0 $\rightarrow$ GO INTO PRODUCTION)

EX. $H_0 : \theta = 500 = \theta_0$ α RISK = 0.05

$H_1 : \theta = 200 = \theta_1$ β RISK = 0.10



HAVE TO FORCE O.C. CURVE THRU THESE POINTS

HOW MANY FAILURES WE GOTTA OBSERVE?

WELL...

$$\frac{\chi^2_{\beta, 2r}}{\chi^2_{1-\alpha, 2r}} = \frac{\theta_0}{\theta_1}$$

FOR PROBLEM AT HAND

$$\frac{\chi^2_{0.1, 2r}}{\chi^2_{0.95, 2r}} = \frac{5}{2} = 2.5$$

COMPARING RATIOS OF COLUMNS IN χ^2 TABLE (

$\Longrightarrow$ $2r = 22$ OR $r = 11$

ii. FOR TIME TERMINATED TEST

FORMAT: $H_0 : \theta = \theta_0$; α RISK

$H_1 : \theta < \theta_0$

TEST STATISTIC = r = # FAILURES

ACCEPT IF $r < r_0$

— FOR REPLACEMENT TEST

r_0 MUST SATISFY

$$\sum_{k=0}^{r_0 - 1} \frac{1}{k!} \left(\frac{nT}{\theta_0} \right) e^{-\frac{nT}{\theta_0}} = 1 - \alpha$$

(MAY EMPLOY THORNDIKE CHART)

— FOR NON-REPLACEMENT TEST

r_0 MUST SATISFY

$$\sum_{k=0}^{r_0 - 1} \binom{n}{k} \left[1 - e^{-\frac{T}{\theta_0}} \right]^{k} \left[e^{-\frac{T}{\theta_0}} \right]^{n-k} = 1 - \alpha$$

(SEE TEXT - IEELL ASLEEP)

b. NORMAL DISTRIBUTION (TEST OF HYPOTHESIS)

ONLY FOR NON-REPLACEMENT

n ITEMS TESTED: ALL FAIL

– CONCERNING μ

$H_0: \mu \overset{\geq}{=} \mu_0 \qquad \alpha$

$H_1: \mu < \mu_0$

TEST STATISTIC: $t_{n-1} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

ACCEPT H_0 IF

$$t_{n-1} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} > t_{1-\alpha, n-1}$$

PG IV 79

EXAMPLE: $n = 4$

$H_0: \mu = \mu_0 = 45$ HRS $\qquad \alpha = 0.05$

$H_1: \mu < \mu_0 = 45$ HRS

TEST STATISTIC

$$t_{n-1} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{\bar{x} - 45}{s/2}$$

WILL ACCEPT IF

$$t_{n-1} = \frac{\bar{x} - 45}{s/2} > t_{0.95;3} = -2.353$$

TEST GIVES $\bar{x} = 43$, $s = 2.56$

$\Rightarrow t_3 = -1.55$

$\Rightarrow$ ACCEPT

— CONCERNING σ

WE WOULD LIKE A SMALL σ.

$H_0 : \sigma \leq \sigma_0$ α

$H_1 : \sigma > \sigma_0$

TEST STATISTIC : $\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma_0^2}$

ACCEPT H_0 IF

$$\chi^2_{n-1} \leq \chi^2_{\alpha;n-1}$$

GIVEN α & β, REQUIRED SAMPLE SIZE IS GIVEN THRU: $\frac{\chi^2_{\alpha,n-1}}{\chi^2_{1-\beta,n-1}} = \frac{\sigma_1^2}{\sigma_0^2}$

C. WEIBULL (WON'T BE ON TEST)

n ITEMS, ALL FAIL; NON-REPLACEMENT

$H_0 : \beta = \beta_0$ α

$H_1 : \beta \neq \beta_0$

TEST STATISTIC : $Z = \frac{\hat{\beta} - \beta_0}{\sqrt{V(\hat{\beta})}}$

$V(\hat{\beta})$ (HAIRY) IS GIVEN ON IV-67

$\hat{\beta}$ IS GIVEN THRU MAXIMUM LIKELIHOOD METHOD

ACCEPT IF

$$-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}$$

— β KNOWN (TIME TERMINATED TEST)

ON pp IV-124-5

6. ACCEPTANCE LIFE TESTING

EXPONENENTIAL - H-108
WEIBULL - TR-3, 4, 6
NORMAL - MIL-STD 414
NON PARAMETRIC - MIL-STD 105 D

a. EXPONENTIAL

H 108 - FOR EXPONENTIAL

CONTAINS PLANS FOR

- FAILURE TERMINATED TESTING
- TIME " "
- SEQUENTIAL TESTING

} WITH & W/O REPLACE.

1. FTT (FAILURE TERMINATED TEST)
   - a. θ_0, r, α
   - b. $\theta_0, \theta_1, \alpha, \beta$
2. TTT
   - a. θ_0, r, α
   - b. $\theta_1, \theta_0, \alpha, \beta$ (w/o Replace)
   - c. $T, \alpha, \beta, P_0, P_1 \ni P =$ FAILED PROPORTIONS OF LOT
   - d. $\alpha, \beta, T, \lambda_0, \lambda_1$ $(\theta_0 = \lambda_0, \theta_1 = \lambda_1)$
3. SEQ.
   - a. $\alpha, \theta_0, \theta_1$ $(\beta = 0.1)$

H108, SEC 2BI (EXPONENTIAL - GIVEN θ_0, r, α)

FAILURE TERMINATED TESTING

I.E. $P_a = P[\theta \geq 1000] = 0.9 \Rightarrow \alpha = 0.10$

$\theta_0 = 1000$

$r = 5$

$H_0 : \theta = 1000 \qquad \alpha = 0.1$

$H_1 : \theta < 1000$

TEST STATISTIC: $\hat{\theta}$

ACCEPT H_0 IF
(FROM TABLE 2B-1) $\hat{\theta} > (\frac{C}{\theta_0}) \times \theta_0$

$> (0.487)(1000)$

> 487 HRS

O.C. CURVE ON TABE 2A-2, PG 2.14

GIVEN $\theta_0, \theta_1, \alpha$, AND β

SEC. 2B-III PG. 2.39

$H_0 : \theta = 900 = \theta_0 \qquad \alpha = 0.05$

$H_1 : \theta = 300 = \theta_1 \qquad \beta \; 0.10$

TEST STATISTIC - $\hat{\theta}$

→ USE TABLE 2B-5, PG 2.41

1. COMPUTE $\frac{\theta_1}{\theta_0}$

2. FROM TABLES, FIND $\frac{C}{\theta_0}$, r

ACCEPT H_0 IF $\hat{\theta} > (\frac{C}{\theta_0})\theta_0$

FOR EXAMPLE

$r = 8$, $\frac{C}{\theta_0} = 0.498$

$\therefore$ ACCEPT H_0 IF $\hat{\theta} > (.498)(900) = 448$

H108, SEC 2BI (EXPONENTIAL - GIVEN θ_0, r, α)

FAILURE TERMINATED TESTING

1a. $P_a = P[\theta \geq 1000] = 0.9 \Rightarrow \alpha = 0.10$

$\theta_0 = 1000$

$r = 5$

$H_0: \theta = 1000 \qquad \alpha = 0.1$

$H_1: \theta < 1000$

TEST STATISTIC: $\hat{\theta}$

ACCEPT H_0 IF
(FROM TABLE 2B-1) $\hat{\theta} > (\frac{C}{\theta_0}) \times \theta_0$

$\geq (0.487)(1000)$

> 487 HRS

O.C. CURVE ON TABLE 2A-2, PG 2.14

GIVEN $\theta_0, \theta_1, \alpha$, AND β

SEC. 2B-III PG. 2.39

$H_0: \theta = 900 = \theta_0 \qquad \alpha = 0.05$

$H_1: \theta = 300 = \theta_1 \qquad \beta\ 0.10$

TEST STATISTIC - $\hat{\theta}$

→ USE TABLE 2B-5, PG 2.41

1. COMPUTE $\frac{\theta_1}{\theta_0}$
2. FROM TABLES, FIND $\frac{C}{\theta_0}$, r

ACCEPT H_0 IF $\hat{\theta} > (\frac{C}{\theta_0})\theta_0$

FOR EXAMPLE

$r = 8, \ \frac{C}{\theta_0} = 0.498$

$\therefore$ ACCEPT H_0 IF $\hat{\theta} > (.498)(900) = 448$

TT.T

GIVEN Θ_0, α, r

SEC. 2C I OF H-108

$H_0: \Theta = \Theta_0 = 1000$ $\alpha = 0.1$

$H_1: \Theta < \Theta_0$ $r = 5, n = 10$

TEST STATISTIC

ACCEPT IF $r < r_0$ AT TIME T

GOTTA FIND r_0, T

FROM TABLE 2C-1(c) WE GET $\frac{T}{\Theta_0}$ & α

$\frac{T_0}{\Theta} = 0.314 \Rightarrow T = 314$

WITH REPLACEMENT $T = 243$ HRS

TTT

GIVEN $\Theta_0, \Theta_1, \alpha, \beta, T$

TEST STATISTIC: r

$H_0: \Theta = \Theta_0$ α

$H_1: \Theta = \Theta_1$ $\beta = 0.1$

$T = 500$ $\Theta_0 = 10^4$ HRS, $\alpha = 0.1$

$\Theta_1 = 2000$

FROM TABLE 2C-3 Pg 252

$\frac{T}{\Theta_0} = \frac{500}{10^4} = \frac{1}{20}$

$\frac{\Theta_1}{\Theta_0} = \frac{1}{5} \Rightarrow n = 23, r = 3$

7. SEQUENTIAL PLAN

MIGHT ALLOW A DECISION TO BE REACHED MORE QUICKLY THAN FTT OR TTT

RULES:

EACH TIME A FAILURE OCCURS,

DECIDE WHETHER TO

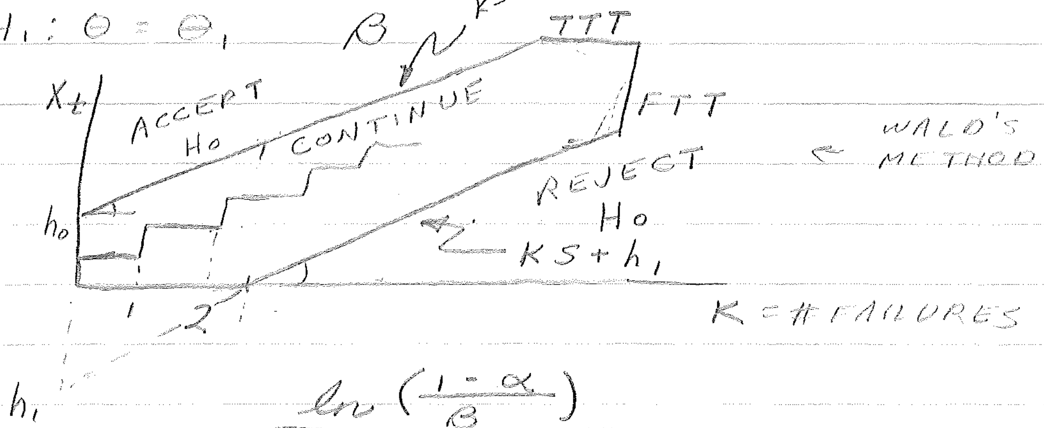
a. ACCEPT & STOP TESTING

b. REJECT & STOP TESTING

c. CONTINUE TESTING

$H_0 : \Theta = \Theta_0$ α (H-108 $\overset{\beta = 0.10}{\text{VARIES } \alpha}$)

$H_1 : \Theta = \Theta_1$ β



$$h_0 = \frac{\ln\left(\frac{1-\alpha}{\beta}\right)}{\frac{1}{\Theta_1} - \frac{1}{\Theta_2}}$$

$$h_1 = \frac{\ln\left(\frac{\alpha}{1-\beta}\right)}{\frac{1}{\Theta_1} - \frac{1}{\Theta_0}}$$

$$S = \text{SLOPE} = \frac{\ln(\Theta_0/\Theta_1)}{\frac{1}{\Theta_1} - \frac{1}{\Theta_0}}$$

(COVERED IN H-108)

PROB. 29, Pg IV-250

$H_0: \theta = \theta_0 = 500 \quad \alpha = 0.05$

$H_1: \theta = \theta_1 = 100 \quad \beta = 0.1$

H: 108; TABLE 2D-1 (a)

USE PLAN B-4

$r_0 = 12, \ h_0/\theta_0 = 0.58, \ \frac{s}{\theta_0} = 0.4086$

$h_1/\theta_0 = -.7453,$

THEN

$h_0 = 290$

$h_1 = 372.$

$s = 204$

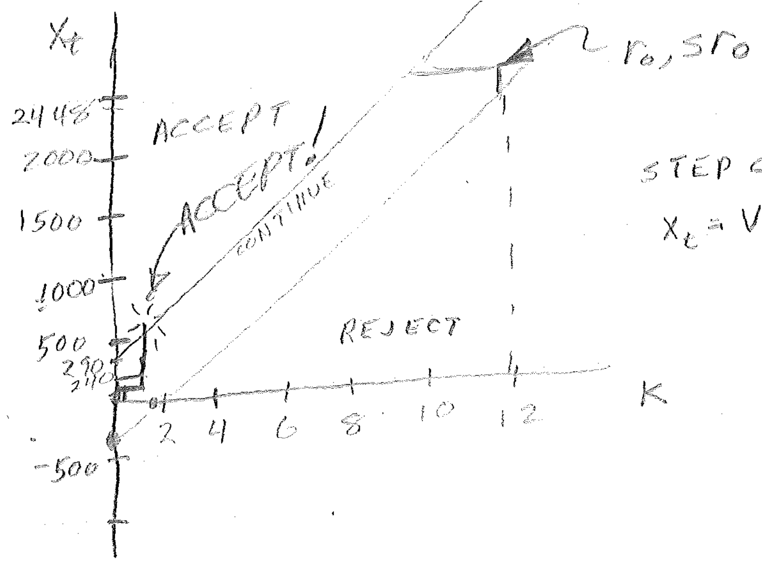
$V_0(t) = h_0 + ks$

$= 290 + 204k$

$V_1(t) = h_1 + ks$

$= -372 + 204k$

$r_0 = 12, \ sr_0 = 2448$ LET $n = r_0$



STEP CURVE FROM

$X_t = V(t) = \sum_{i=1}^{r} X_i + (n-r)X_r$

b. WEIBULL TR-3

"SAMPLING PROCEDURES AND TABLE_ FOR LIFE AND RELIABILITY TESTING BASED ON WEIBULL DISTRIBUTION (MEAN LIFE CRITERION)"

TR-3 - MEAN LIFE CRITERION ~ $E[x] = n\Gamma(1+\frac{1}{\beta})$

TR-4 - HAZARD RATE CRITERION ~ $Z(t)$

TR-6 - RELIABLE LIFE CRITERION ~ ρ

ALL OF THESE DOCUMENTS ASSUME:

β IS KNOWN
α RISK = 0.05
β RISK = 0.10
TTT WITHOUT REPLACEMENT

→ IN TR-3

$H_0: \mu = \mu_0$ $\alpha = 0.05$
$H_1: \mu = \mu_1$ $\beta = 0.10$ $(\mu_0 > \mu_1)$
t = TEST TERMINATION TIME

RULES:

1. ENTER TABLES 3(a, b, c...) WISH SHAPE PARAMETER (β) AND (t/μ_1) → USE LOWER NEXT RATE
IF β IS NOT ENTERED, USE NEXT BIGGER
2. GO DOWN THE APPROPRIATE COLUMN UNTIL t/μ_0 → OR NEXT HIGHER IS ENCOUNTERED IN PARENTHESES. THE # ABOVE t/μ_0 IS THE SAMPLE SIZE n
3. IN THE L.H. COLUMN ⇒ C = ACCEPTANCE #.
ie, ACCEPT H_0 IF $r \leq C$

EXAMPLE: Pg IV-182

$\beta = 1\frac{2}{3}$

$H_0: \mu = \mu_0 = 4000$ $\alpha = 0.05$ TTT

$H_1: \mu = \mu_1 = 2000$ $\beta = 0.10$ $t = 1000$

$$100 \frac{t}{\mu_0} = 100 \left(\frac{10^3}{4000}\right) = 25$$

$$100 \frac{t}{\mu_1} = 100 \left(\frac{10^3}{2000}\right) = 50$$

Pg 31 in TR-3 $\Rightarrow$ $n = 44$ ITEMS

ACCEPT IF $r \leq 6 = C$

→ IN TR-4 :

$H_0: Z(t) = Z_0(t)$ $\alpha = 0.05$

$H_1: Z(t) = Z_1(t)$ $\beta = 0.10$

t = TEST TIME (TERMINATION)

ACCEPT H_0 IF $r < C \ni C$ IS TABLED

(TABLES 3(a, b, c, ...))

- FIND APPROPRIATE (GIVEN) β
- FIND $t Z_1(t) \times 100$ → COLUMN
- FIND $t Z_0(t) \times 100$ → IN PARENTHESIS
- READ n, C

EXAMPLE: Pg IV-184

GIVEN $\beta = 2/3$ t = 30 DAYS = 720 HRS

$H_0: Z = Z_0 = 0.0002$ $\alpha = 0.05$

$H : Z = Z_1 = 0.00015$

$100 t Z_1 = 10.80$ $100 t Z_0 = 1.44_0$

FROM TABLE 3C, $n = 37$, $r = 2$

→ IN TR-6

IF 90% OF TESTED ITEMS ARE EXPECTED
TO SURVIVE BEYOND 50,000 MILES
H_0: $\rho = \rho_0$ $\alpha = 0.05$? ρ = RELIABLE LIFE
H_1: $\rho = \rho_1$ $\beta = 0.10$ $[\rho_0 > \rho_1]$

TABLES SET UP FOR $r = 0.5, 0.9, 0.99$
(USE TABLES 3(a, b, ...))

Ex. Pg IV-190
$\beta = 2$ $t = 2160$ HRS, $r = 0.9$
H_0: $\rho = \rho_0 = 5000$ $\alpha = 0.05$
H_1 $\rho = \rho_1 = 2500$ $\beta = 0.10$
FIND TABLE 3b7, pg 50
$\frac{t}{\rho_1} \times 100 = 86.4$ ← USE 80
$\frac{t}{\rho_0} \times 100 = 43.2$ ← USE 44
$\Rightarrow n = 161, c = 6$

9 = 2 4225

C. NORMAL DISTRIBUTION - USE MIL STD. 414

SEARCH O.C. CURVES FOR AN ADEQUATE PLAN

[TABLES A-3]

GOTTA FIND SAMPLE CODE LETTER

ENTER TABLE B-1 TO FIND n AND

ACCEPTANCE CRITERIA

MAY TEST USEING:

- RANGE METHOD
- σ UNKNOWN METHOD ← WE ARE CONCERNED WITH THIS ONE (SEE B, Pg 35
- σ KNOWN "

EXAMPLE: A TIRE IS CONSIDERED ACCEPTABLE IF ITS R[20,000 MILES] = 0.95 AND UNACCEPTABLE IF ITS R[20,000] = 0.6. FOR α AND β RISKS < 0.10, FIND ~~THE~~ MINIMUM SAMPLE SIZE PLAN

H_0: $R[20,000] = R_0 = 0.95$ $\alpha < 0.1$

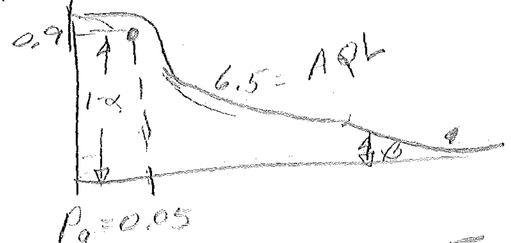
H_1: $R[20000] = R_1 = 0.6$ $\beta < 0.1$

O.C. CURVES ARE GIVEN IN % DEFECTIVE

% DEFECTIVE = $1 - R_0 = 0.05$

$1 - R_1 = 0.40$

LOOK FOR CURVE:



ON Pg 9, USE "6.5" CURVE ⇒ CODE E

TABLE B-1, Pg 39

$n = 7, K = 0.95$ (CONT.)

1. WE TEST 7 ITEMS UNTIL ALL FAIL,
KEEP TRACK OF MILLAGES ; $K = 0.95$

FAILURE TIMES:

19,000
19,700
20,100
20,500
21,500
21,300
22,200
28,000

SEE Pg 37-8 WE TALLY

1. $n = 7$
2. $\sum X_i = 150,800$
3. SUM OF SQUARES (SS) $\sum X_i^2 = 3,303,880,000 = SS$
4. CORRECTION FACTOR (CF): $\frac{1}{n}\left[\sum X\right]^2 = 3,248,662,8.$
5. CORRECTED SUM OF SQUARES (SS): $\sum X^2 - CF = 55,217,14$
6. VARIANCE $= \frac{SS}{n-1} = 9,202,857.167$
7. STANDARD DEVIATION: $\sqrt{V} = 3,033.6$ MILES
8. MEAN $= \frac{1}{n} \sum X = 21,554.9$
9. $L = 20,000$
10. $\frac{\bar{X} - L}{S} = 0.5086$
11. $K = 0.955$
12. IS $\frac{\bar{X} - L}{S} \geq K$? NO!
$\Rightarrow$ REJECT H_0

B. TABLES B-5
0.51 FOR $n = 7 \Rightarrow 31.39\%$ MAXIMUM DEFECTIVE
SEE TABLES A1 & 2, Pg 4

d. PARAMETRIC

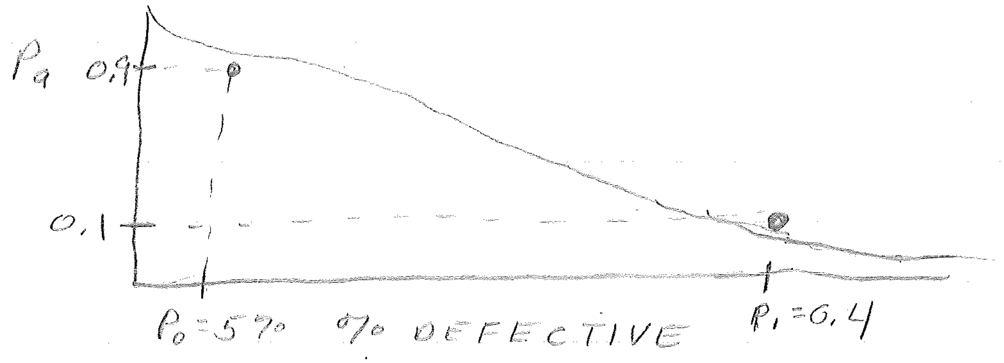
MIL-STD. 105-D

$H_0 : R(20{,}000) = 0.95$ $\alpha = 0.1$

$H_1 : R(20{,}000) = 0.6$ $\beta = 1$

(ASSUME DISTRIBUTION NOT KNOWN)

TIME TERMINATED TEST



IDENTIFY

- SAMPLE CODE LETTER
- AQL

FOR PROBLEM AT HAND

CODE D, AQL = 6.5

$n = 8$ ITEMS TESTED FOR 20,000 MILES

ACCEPT H_0 FOR $r \leq 1$ = AC

REJECT H_0 FOR $r > 2$ = REJ

e. ACCELERATED LIFE TEST (MIL HNDBOOK 217)

TESTING UNDER USE CONDITIONS TO DATE.

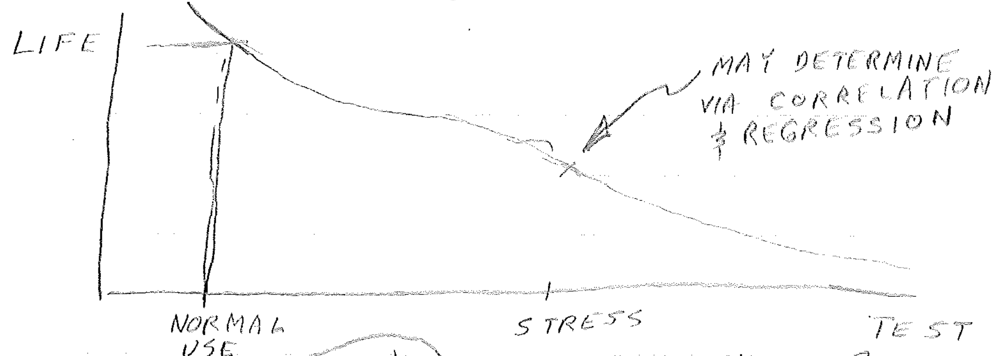
WHAT ABOUT TEST CONDITIONS OTHER THAN USE CONDITIONS

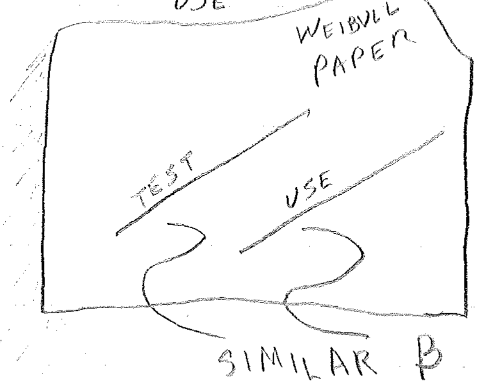
1. TEST CONDITIONS MORE SEVERE THAN OPERATIONAL CONDITIONS (OVERSTRESS TESTING)
2. OPERATIONAL CONDITIONS MORE SEVERE THAN TEST CONDITIONS

EXAMPLE: LIGHTBULB - WHAT FACTORS INFLUENCE LT. BLB.?

1. THERMAL SHOCK
2. VIBRATION
3. APPLIED CURRENT
4. CYCLING
5. PRESSURE

WE DO ACCELERATED TESTING





ACCELERATION FACTOR

$$= \frac{\mu_{USE}}{\mu_{TEST}}$$

NO CORRELATION OR REGRESSION COVERED

STRESS- STRENGTH ANALYSIS

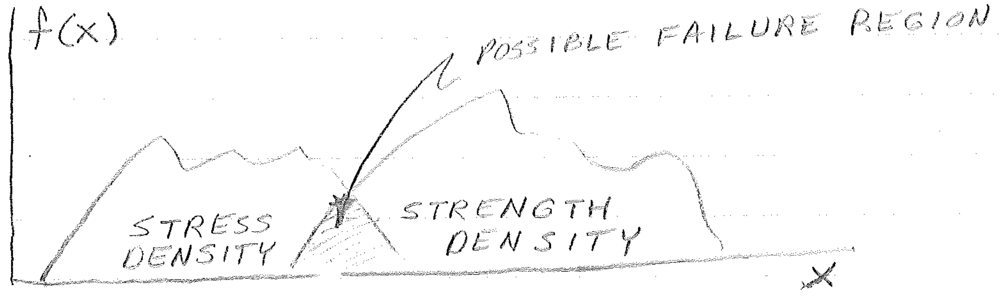
(OR PROBABILISTIC METHODS IN RELIABILITY DESIGN)

USED IF: STRESS CONDITIONS ARE KNOWN
: MATERIAL CHARACTERISTICS (STRENGH) IS KNOWN

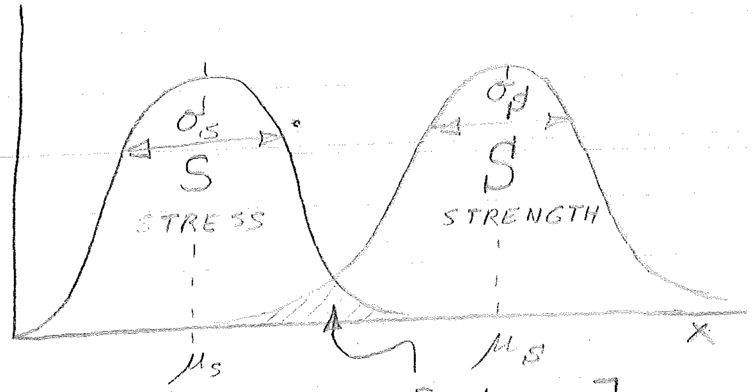
RELIABILITY PREDICTION TECHNIQUES

1. NORMAL DISTRIBUTION ASSUMED FOR BOTH STRESS AND STRENGTH
2. SAFETY MARGIN TECHNIQUE (APPROXIMATION)
3. GENERAL OR GRAPHICAL METHOD

IN GENERAL



A. NORMAL STRENGTH-STRESS DISTRIBUTIONS



$P[\text{FAILURE}] = P[S' \cap s]$

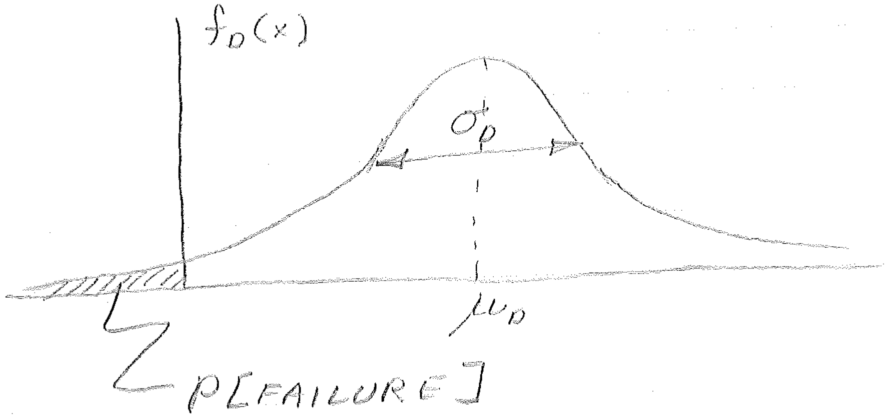
LET $D = \mu_{S'} - \mu_s$

$$\text{IF} \begin{cases} D > 0 & \text{WE HAVE SUCCESS} \\ D \leq 0 & \text{" " FAILURE} \end{cases}$$

ADDING TWO NORMAL POPULATIONS GIVES AN NORMAL POPULATION WITH

MEAN: $\mu_D = \mu_{S'} - \mu_s$

VARIANCE: $\sigma_D^2 = \sigma_{S'}^2 + \sigma_{s}^2$



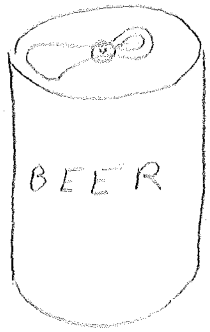
$\frac{\mu_D}{\sigma_D}$ = RELIABILITY MARGIN

$$\therefore P[\text{FAIL}] = p[D < 0] = P\left[Z \leq \frac{-\mu_D}{\sigma_D}\right]$$

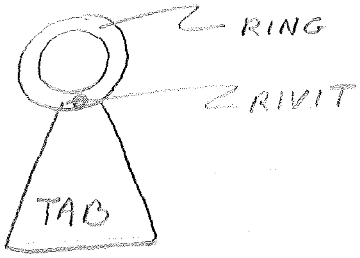
$$= P\left[Z \leq \frac{\mu_s - \mu_{S'}}{\sqrt{\sigma_s^2 + \sigma_{S'}^2}}\right]$$

$$R = 1 - P\left[Z > \frac{\mu_D}{\sigma_D}\right]$$

EXAMPLE:



FAILURE MODE: RIVET TEARS BEFORE TAB IS COMPLETELY REMOVED



TAB: $\mu_S = 2^{\#}$ $\sigma_S = 0.4^{\#}$

RIVIT: $\mu_{\overset{\cancel{S}}{F}} = 3^{\#}$ $\sigma_S = 0.3^{\#}$

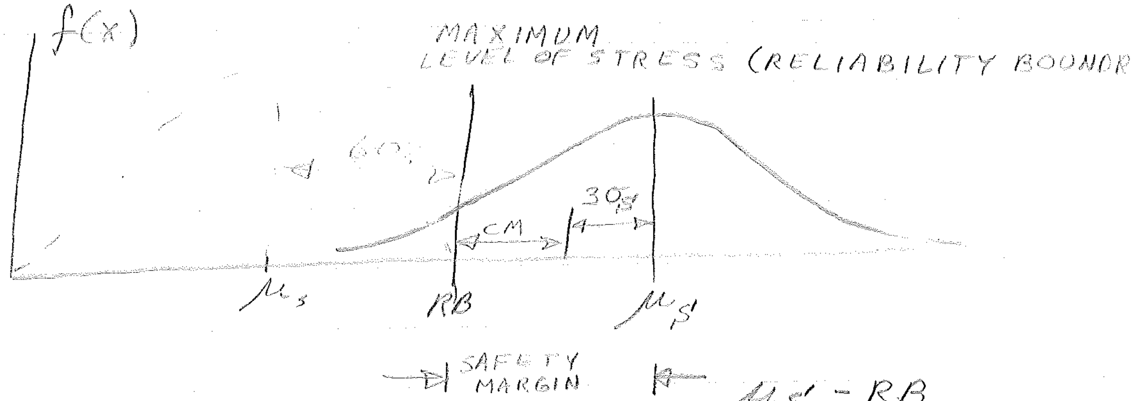
RELIABILITY MARGIN: $\frac{\mu_D}{\sqrt{\sigma_D}} = \frac{\mu_S - \mu_F}{\sqrt{\sigma_S^2 + \sigma_F^2}} = \frac{1}{0.5} = 2$

$P_F = P[Z > 2] = 0.02275$

$R_1 = P_A = 0.97725$

B. RELIABILITY BOUNDRY (SAFETY MARGIN)

CONSIDERS MAXIMUM LEVEL OF STRESS WE ARE LIKELY TO ENCOUNTER



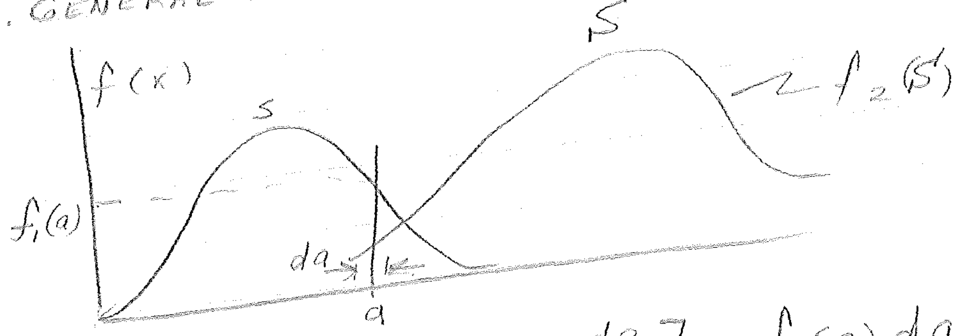
SAFETEY MARGIN = $SM = \frac{\mu_{S'} - RB}{\sigma_{S'}}$

= # OF STANDARD DEVIATIONS RB IS FROM $\mu_{S'}$

SOMETIMES: $RB = \mu_S + 6\sigma_S$

CONTINGINCY MARGIN: $CM + 3\sigma_{S'} = SM$

C. GENERAL METHOD



$$P\left[a-\frac{da}{2}\le a\le a+\frac{da}{2}\right]=f_1(a)\,da$$

$$P[S'>s]=P[S'>a]=\int_a^\infty f_2(S')\,ds$$

$$=1-F_2(S')\Big|_a^\infty$$

$$=R_2(a)$$

$$R=\int_{-\infty}^{\infty} f_1(a)\left[\int_a^\infty f_2(S')\,dS'\right]da$$

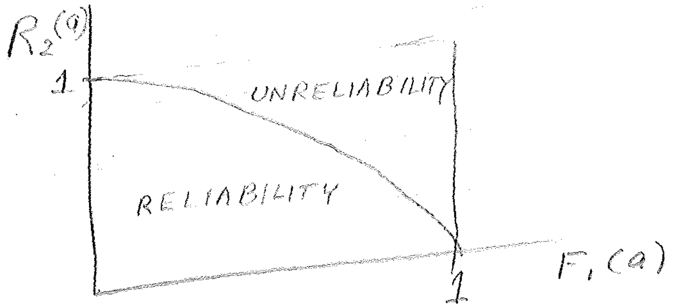
$$=\int_{-\infty}^{\infty} f_2(a)\left[\int_{-\infty}^{a} f_1(s)\,ds\right]da$$

$$=\int_0^1 R_2(a)\,dF_1(a) \Leftarrow \text{EXACT SOLUTION}$$

$$\ni F_1(a)=\int_{\infty}^{a} f_1(a)\,da$$

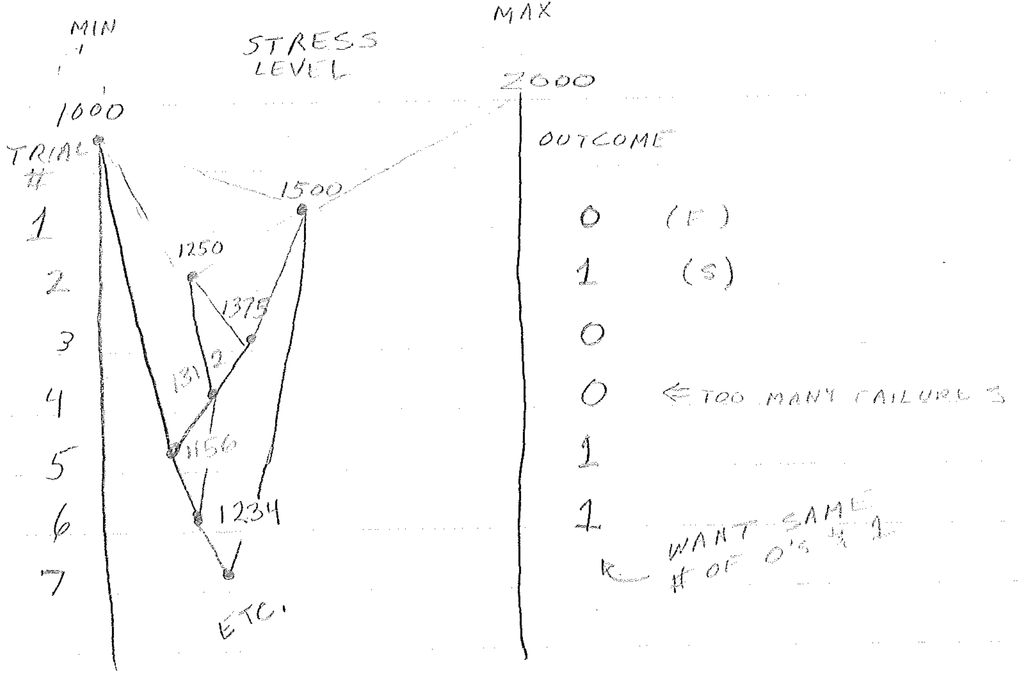
$$\frac{dF_1(a)}{da}=f_1(a)$$

$$dF_1(a)=f_1(a)\,da$$



ONE SHOT ITEMS

ASSUME NORMAL DISTRIBUTION (SEE pg 22)



MAINTAINABILITY

MAINTAINABILITY IS A CONSIDERATION IN DESIGN
MAINTANENCE IS A CONSEQUENCE OF DESIGN

MAINTAINABILITY INDICES -

1. M → PROBABALISTIC MEASURE
2. $\bar{M}$ = MTTR = MEAN TIME TO REPAIR
3. $\tilde{m}$ = THE NUMBER OF X HRS NEEDED FOR ALL MAINTANENCE TAST

EX: $\tilde{m}_{50} \Rightarrow$ 50% OF REPAIRS DONE IN $\tilde{m}$ HRS

$P[\underline{X} < x] =$

$=$ P[COMPLETED MAINTENANCE TASK WITH TIME X]

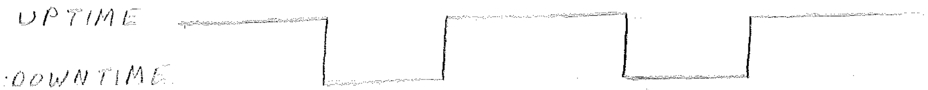
MAINTENANCE INDEX

$$M \text{ RATIO} = \frac{\text{TOTAL MAINTANENCE MAN HRS.}}{\text{UNIT OF OPERATION}}$$

EX: UNIT OF OPERATION = 100,000 MILES

2 POSSIBLE MAINT. ACTIONS

1. PREVENTIVE MAINTAINANCE
2. CORRECTIVE " — REQUIRED AS A RESULT OF SYSTEM FAILURE



(ACTIVE (PEOPLE WORKING ON IT)
(AND DELAY (BROKEN, WAITING FOR PARTS)

— A_i = INHERENT AVAILABILITY $= \dfrac{\text{UPTIME}}{\text{TOTAL TIME} - \text{DELAY TIME}}$

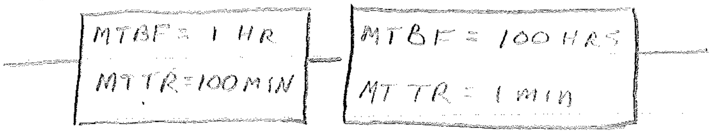
$= \dfrac{MTTF}{MTTF + MTTR}$

[ASSUMES PERFECT SUPPLY & SUPPORT
CONSIDERS ONLY CORRECTIVE MAINT.]

— A_a = ACHEIVED AVAILABILITY $= \dfrac{MTBM}{MTBM + MTTR_{ACTIVE}}$

— A_o = OPERATIONAL AVAILABILITY $= \dfrac{MTBT}{MTBM + MTTR_{(ACTIVE \,\&\, DELAY)}}$

EXAMPLE:



CONSIDER A 100 HR PERIOD.

$MTTR_{SYS} = \dfrac{\text{TOTAL TIME SPENT REPAIRING}}{\text{TOTAL \# REPAIR ACTIONS}} = \dfrac{(100)(100)+(1)(1)}{101}$

$\cong 100$ MINUTES

IF $MTBF_B = 1$ HR, $MTTR = \dfrac{(100)(100)+(100)(1)}{200} \cong 50.5$ MINUTES

PROBABALISTIC ASPECTS OF $\overline{M}$

A SYSTEM REQUIRES PERFORMANCE OF A MAINTAINANCE ACTION. MAINTAINANCE ACTION BEGINS AT TIME $X=0$ AND FINISHES AT TIME $\underline{X}$.

SOME DISCRIPTIVE DISTRIBUTIONS ARE

1. LOG NORMAL
2. GAMMA w / $\alpha = 2$
3. WEIBULL w/ $\beta = 2$
4. EXPONENTIAL (NOT SO HOT)
5. NORMAL
6. TRUNCATED NORMAL

COCOA STAINS

1. LOG NORMAL

$$f(x) = \frac{1}{x\rho\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\frac{\ln x - \omega}{\rho}\right]^2}$$

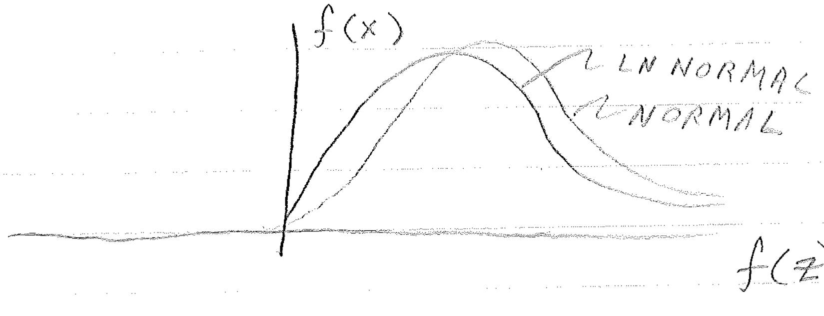
$\mu(x)$

OR $\ln X$ IS NORMALLY DISTRIBUTED

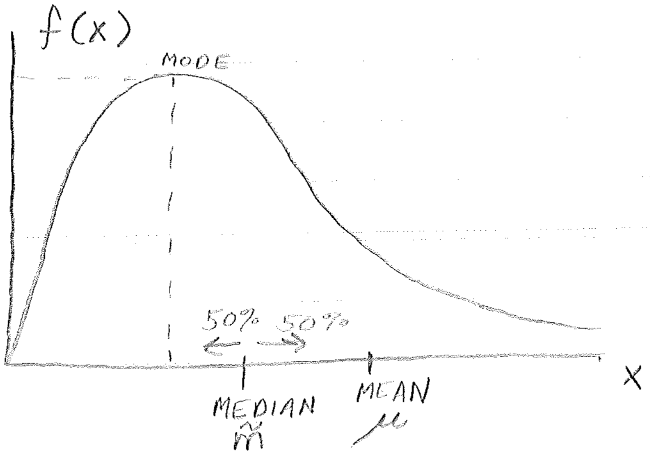
$\omega = 0 \Rightarrow \mu = 1.65$

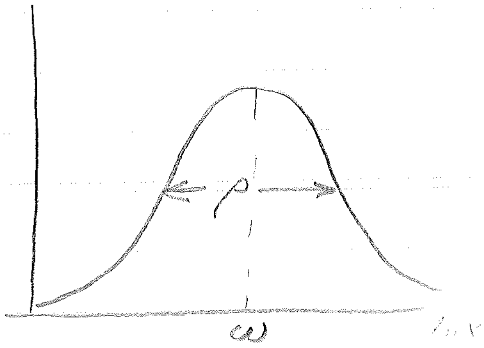
$\rho = 1 \Rightarrow \sigma = 2.16$

IF THE RANDOM VARIABLE $\underline{X}$ IS LOG NORMALLY DISTRIBUTED WITH MEAN μ AND STANDARD DEVIATION $\sigma_{\ln X}$, THEN $\ln \underline{X}$ IS NORMALLY DISTRIBUTED WITH MEAN ω AND STD. DEVIATION ρ



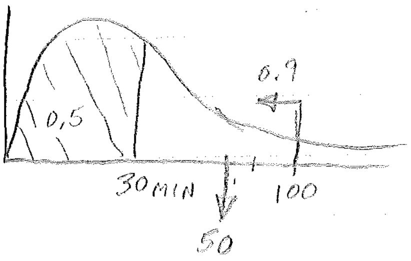
$$f(z) = f\left[\frac{\ln x - \omega}{\rho}\right]$$





$$\tilde{m} = e^{\omega}$$
$$\mu = e^{\omega + \frac{1}{2}\rho^2}$$
$$m(x) = P[\underline{X} < x] = P\left[Z < \frac{\ln x - \omega}{\rho}\right]$$

EXAMPLE: A SYSTEM HAS HAD ITS MAINTAINANCE TIMED ANALYZED. THE MEADIAN TIME TO RESTORE $\tilde{m} = 30$ MIN & . $\tilde{m}_{MAX\ 0.9} = 100$ MIN. ASSUME LOG NORMAL DISTR. FIND m(50 min)



$$\omega = \ln \tilde{m} = 3.4$$
$$\ln 100 = 4.61 =$$
$$P\left[Z < \frac{\ln 100 - \omega}{\rho}\right] = 0.90$$
$$P\left[Z < \frac{1.21}{\rho}\right] = 0.96$$
$$Z_{0.9} = 1.28 \Rightarrow \rho = 0.945$$
$$m(50) = P[\underline{X} < 50] = P\left[Z < \frac{\ln 50 - \omega}{\rho}\right]$$
$$= P\left[Z < \frac{3.91 - 3.40}{0.945} = 0.54\right] = 1 - 0.295 = 0.705$$

M APPORTIONMENT (Pg VI-32)

SIMILAR TO ARING METHOD

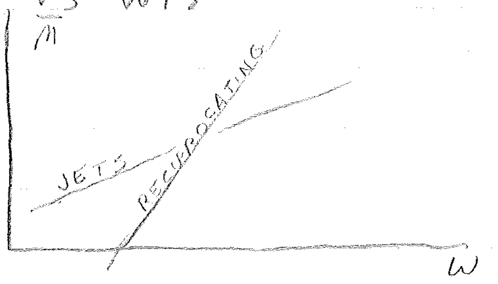
[MIL-HDBK-472 → MAINTAINABILITY]

WE WOULD LIKE A METHOD

1. REFLECTS OPERATIONAL REQUIREMENTS
2. UNIVERSALLY APPLICABLE
3. REASONABLE MIX TWIXT COST TO APPLY METHOD AND ITS ACCURACY
4. EZ TO APPLY
5. EASILY OBTAINABLE DATA

INFORMAL METHODS

1. SWAG → FAIRLY GOOD IF YOU GOT CAPABLE CWO'S, ENS, ETC.
2. M VS WT.



MIL. HDBK-472 MAINTAINABILITY PREDICTION HANDBOOK

RCA TECHNIQUE (IN MIL HDBK 472)

GENERAL APPROACH

1. SELECT A SAMPLE SIZE
2. SELECT SPECIFIC EXAMPLE OF TASKS
3. DETERMINE TASK TIMES
4. COMBINE INDIVIDUAL TASK TIMES TO YIELD SYSTEM $\underline{M}$ CHARACTERISTICS

EXAMPLE: TASK - CHANGE A TIRE

REMOVE FLAT & REPLACE WITH SPARE

START <u>CHECKLIST</u> A, Pg 62 A_i

1. ACCESS (EXTER.) 4
2. LATCHES & EXTERNAL FASTENERS 0
3. LATCHES & INTERNAL 0
4. ACCESS (INTERN) 2
5. PACKAGING .... 2 $\Sigma A_i = 37$
6. UNITS / PARTS 2
7. VISUAL DISPLAY 4
8. FAULT & OP. IND. (BIT) NA . 4
9. TEST POINTS (AVAIL) .. 3
10. TEST PTS (ID) 2
11. LABELING 2
12. ADJUST. 4
13. TESTING 4
14. PROT. DEVICES 2
15. SAFETY 2

(CONT.)

CHECKLIST B Pg VI-68

1. EXT TEST EQ 4

2. CONNECTORS 4

3. JIGS OR FIXTURES 2

4. VISUAL CONTACT ... NA → 4 $\Sigma B_i = 26$

5. ASSIS (OP. PERSONEL) 4

6. " (TECH) 4

7. " (SUP) 4

$\Sigma A_i = 37$ $\Sigma B_i = 26$

CHECKLIST C Pg VI-71

1 - 2

2 - 2

3 - 3

4 - 4

5 - 4 $\Sigma C_i = 27$

6 - 4

7 - 3

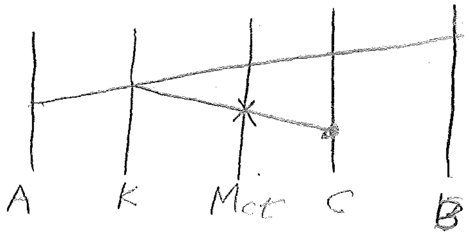
8 - 2

9 - 1

10 - 2

$\Sigma A_i = 37$ $\Sigma B_i = 26$ $\Sigma C_i = 27$

NOMOGRAPH ON Pg VI-46 GIVES



$M_{ct} \simeq$ 35 MINUTES

MIL-STD 471A: $\underline{M}$ DEMONSTRATION
473 (ALTERNATIVE)

471 DEMONSTRATION TEST PLANS

1. LOG-NORMAL; $\overline{M}_{ct}$ (MEAN COR. TIME)
   ; $\tilde{M}_{MAC_{cT}}$ (MAX " ")
   ; SEQUENTIAL TESTS ($n \leq 100$)

2. ANY DISTRIBUTION: $\overline{M}_{ct} \rightarrow$ (1-B CONFIDENCE)
   ; $\overline{M}_{PT}$ $\nearrow$ $\overset{\text{MEAN}}{\text{(PREV.}}$ MAIT TIME)
   ; $M_{MAX_{ct}}$ (LOW CONF. LEVEL)
   $n > 50$

3. LOG-NORMAL; $\tilde{M}_{ct}$ ($\alpha = 0.05$)
   $\hookrightarrow$ EQUIP REPAIR TIME: $n = 20$

4. ANY DISTRIBUTION: $\tilde{M}_{ct}$
   $\tilde{M}_{pt}$
   $M_{MAX\,ct}$ (90%)
   $M_{MAX\,pt}$ (90%)
   } 75 OR 90% CONFIDENT

5. (PT. ESTIMATE)

6. ANY DISTRIBUTION: $\overline{M}_{pt}$ & $M_{MAX_{PT}}$
   (NO RISKS STATED)

SYSTEM EFFECTIVENESS

FUNCTION OF

- AVAILABILITY
- DEPENDIBILITY
- CAPABILITY

SYSTEM EFFECTIVENESS

1. DETERMINISTIC APPLICATION
2. PROBABALISTIC ANALYSIS

DETERMINISTIC APPROACH

(DEVELOPED BY WSEIAC - WEAPON SYSTEM EFFECTIVENESS INDEPENDENT ADVISORY COMMITTEE) SAYS

$$[E] = [A][D][C]$$

MATRIX MATH

$$\underset{\underset{\text{ROW}}{\uparrow}}{a_i}\;\underset{\underset{\text{COLUMN}}{\uparrow}}{j}$$

$$\begin{bmatrix} X_{12} & X_{12} & X_{13} \\ 0 & X_{22} & X_{23} \\ 0 & 0 & X_{33} \end{bmatrix} \Rightarrow \text{TRIANGULAR MATRIX}$$

MULTIPLY

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 13 \\ 11 & 19 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 11 & 19 \\ 8 & 22 \end{bmatrix}$$

$$n_1 \times \underbrace{m_1 \quad\quad n_2}_{\text{MUST BE SAME}} \times m_2$$

$$n_1 \times m_2$$

MULTIPLY

$$\Rightarrow [n \times m][m \times p][p \times q] = [n \times q]$$

— AVAILABILITY MATRIX

$$[A] = \underset{\longleftarrow \text{ STATE } \longrightarrow}{\begin{bmatrix} a_{11} & a_{12} \cdots & \\ a_{21} & & \\ \vdots & & \end{bmatrix}} \updownarrow \text{SYSTEMS} \qquad (q \times n)$$

FOR ONE

$$[A] = [a_{11} \; a_{12} \cdots \quad]$$

a_{ij} = P[SYSTEM i WILL BE IS STATE j OF REPAIR @ START OF MISSION (RANDOM PT. TIME)]

— DEPENDEBILITY MATRIX

$$[D] = \begin{bmatrix} d_{11} & d_{12} \cdots & d_{1n} \\ d_{21} & & \\ \vdots & & \\ d_{n1} & & d_{nn} \end{bmatrix} \Leftarrow \text{SQUARE!} \qquad (n \times n)$$

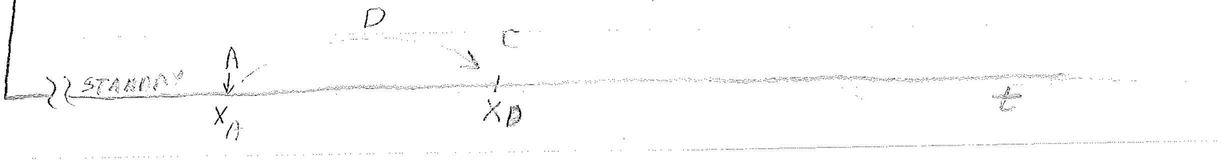
d_{12} = P[END IN STATE OF REPAIR 2 GIVEN IT STARTED IN STATE 1]

— CAPABILITY MATRIX

$$[C] = \begin{bmatrix} c_{11} & c_{12} \cdots & c_{1p} \\ c_{21} & & \\ \vdots & & \\ c_{n1} & \cdots & \end{bmatrix} \qquad (n \times p)$$

C_{12} = P[ACCOMPLISHING OBJECTIVE #2 GIVEN THAT FINISH IN STATE 1]

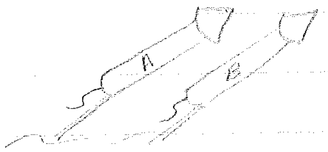
ILLUSTRATION OF A, D, & C

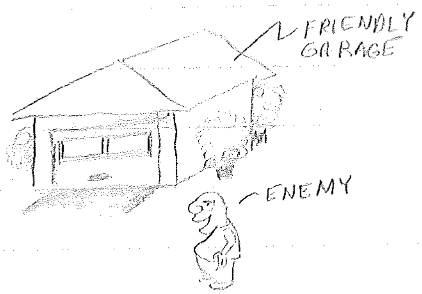


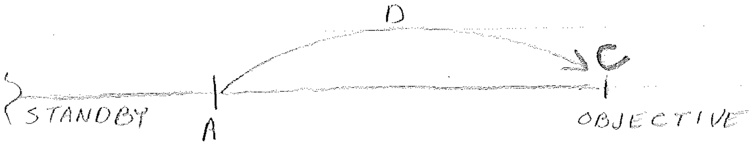
ANALYSIS ASSUMPTIONS

- THE n STATES OF REPAIR MUST BE MUTUALLY EXCLUSIVE
- " " " " " " " DEFINED SO THAT THEY ALWAYS APPLY (ie COVER ALL CONDITIONS)

EXAMPLE







OBJECTIVES: 1. DESTROY ENEMY

2. DO NOT DESTROY GARAGE

APRIORI: - MTBF = 90 HRS/MSSLE

- MTTR = 10 "
- FAILURES OF A & B INDEP
- 2 REPAIR CREWS

DURING FL: - FLIGHT DURATION = X = 1 MIN

- MTBF = 10 MIN/MSSLE
- PERFECT GUIDANCE

(CONT.)

FOR 2 MISSLES

P[DESTROYING ENEMY] = 0.9

P[DESTROYING FRIEND] = 0.5

FOR 1 MISSLE

P[DESTROY ENEMY] = 0.6

P[DESTROY FRIEND] = 0.2

3 STATUSES (STATES)

1. 2 MISSLES
2. 1 MISSLE
3. 0 MISSLE

FIND SYSTEM EFFECTIVENESS

— $A = [a_{11} \ a_{12} \ a_{13}]$

FOR A SINGLE MISSLE = $\frac{MTBF}{MTBF + MTTR} = \frac{90}{90+10} = 0.9 = P[A]$

$\Rightarrow a_{11} = (0.9)(0.9) = 0.81 \quad = P[A]P[B]$

$a_{12} = 2(0.1)(0.9) = 0.18 \quad = 2P[A]P[\bar{B}]$

$a_{13} = (0.1)(0.1) = 0.01$

$\Rightarrow A = [0.81 \ \ 0.18 \ \ 0.01]$ (NOTE $\Sigma a_i = 1$)

— $D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$

$R(1) = e^{-1/10} = 0.9$

$d_{11} = 0.81 \quad d_{12} = 2(0.9)(0.1) = 0.18 \quad d_{13} = 0.01$

$d_{21} = 0 \quad d_{22} = 0.9 \quad d_{23} = 0.1$

$d_{31} = 0 \quad d_{32} = 0 \quad d_{33} = 1$

$$D = \begin{array}{cc} \begin{bmatrix} 0.81 & .18 & .01 \\ 0 & .9 & .1 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{l} \Sigma \rightarrow 1 \\ \Sigma \rightarrow 1 \\ \Sigma \rightarrow 1 \end{array} \end{array}$$

— $$[C] = \begin{bmatrix} C_{11} & C_{21} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{bmatrix}$$

$C_{11} = 0.9 \qquad C_{21} = 0.5$

$C_{12} = 0.6 \qquad C_{22} = 0.8$

$C_{13} = 0 \qquad C_{23} = 1.0$

$$\Rightarrow C = \begin{bmatrix} 0.9 & 0.5 \\ 0.6 & 0.8 \\ 0 & 1.0 \end{bmatrix}$$

$$[E] = [A][D][C]$$

$$= [0.81 \quad 0.18 \quad 0.01] \begin{bmatrix} 0.81 & 0.18 & 0.01 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0.9 & 0.5 \\ 0.6 & 0.8 \\ 0 & 1.0 \end{bmatrix}$$

$$= [0.78 \, , \quad 0.61]$$

HAZARD PLOTTING

EX. TIME FROM START OF TEST time	K	$100(\frac{1}{K}) = \hat{h}(x)$	$\hat{H}(x) = \Sigma \hat{h}(x$
31.7*	16	100/16 = 6.25	6.25 %
39.2*	15	100/15 = 6.67	12.92 %
57.5*	14	100/14 = 7.14	20.06 %
65	13		
65.8*	12	100/12 = 8.33	28.39
70*	11	100/11 = 9.09	37.48
75	10		
75.1	9		
87.5	8		
88.3	7		
94.2	6		
101.7	5		
105.8*	4	$\frac{100}{4} = 25$	62.48
109.2	3		
110*	2	$\frac{100}{2} = 50$	112.48
130	1 REVERSE RANK		

* DENOTES FAILURE

OTHERS: SUSPENSION TIME

WILL TEST FOR WEIBULL: $h(x) = \left(\frac{\beta}{\eta}\right)\left(\frac{x}{\eta}\right)^{\beta-1}$

CUMMULATIVE HAZARD: $H(x) \triangleq \int_{-\infty}^{x} h(x)dx$

FOR WEIBULL $H(x) = \left(\frac{x}{\eta}\right)^{\beta}$

LINEARIZING GIVES $[H(x)]^{1/\beta} = \frac{x}{\eta}$

$$\Rightarrow x = \eta\,[H(x)]^{1/\beta}$$

$$\ln x = \ln \eta + \frac{1}{\beta}\ln H(x)$$

MAY PLOT ON LOG-LOG PAPER WILL GET A STAIT LINE WITH SLOPE $\frac{1}{\beta}$

STEPS IN MAKING A HAZARD PLOT

1. Order the n sample times smallest to largest without regard to censored or failure data.

2. Label the times with reverse ranks. The failure times are marked to distinguish from the censored times.

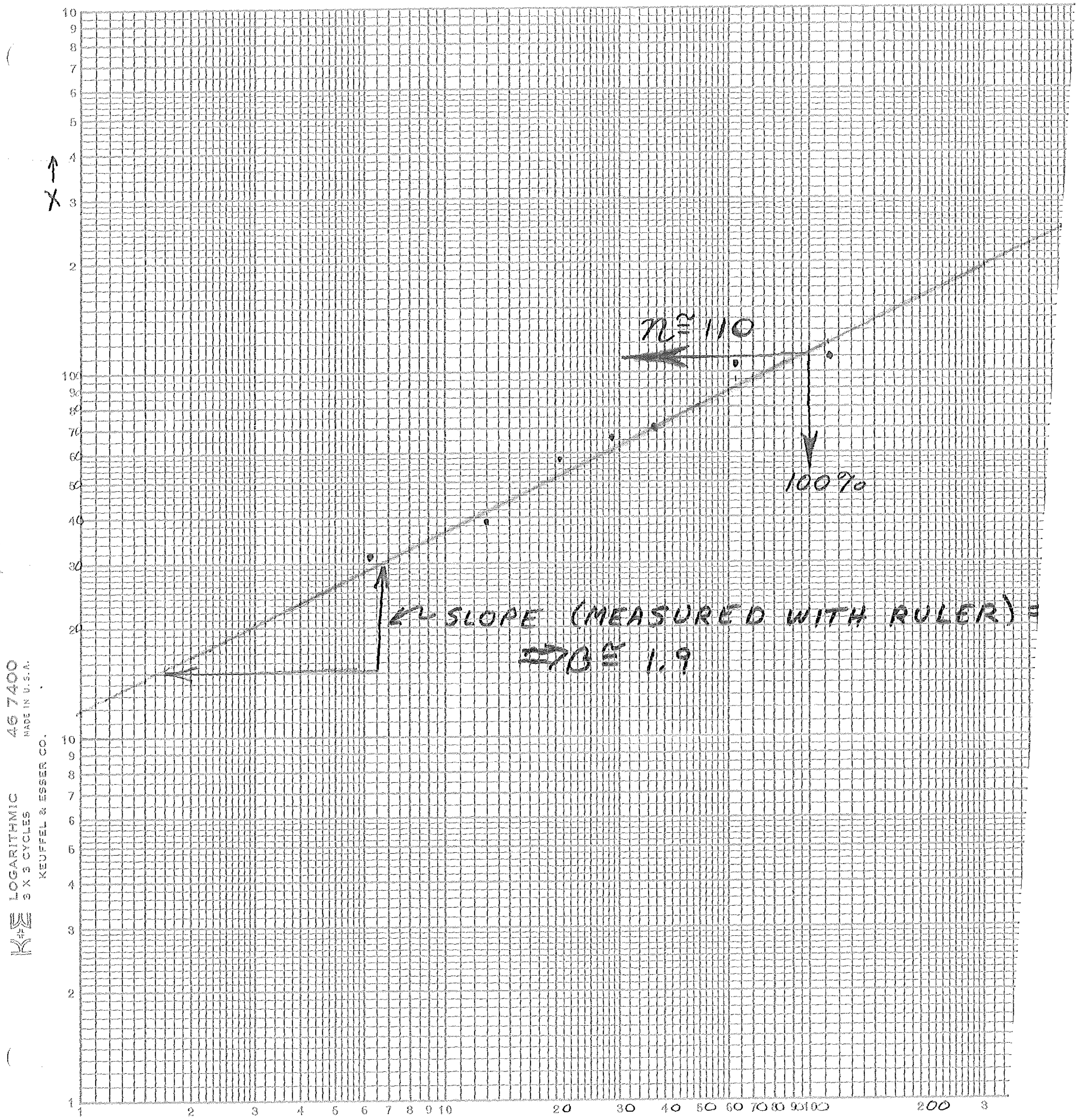
3. Calculate the hazard value for each failure as 100/K, where K is its reverse rank. The hazard value is the observed conditional probability of failure, since 1 out of K units failed in passing through that age. (i.e., the percentage 100 (1/K) of the K units that ran that length of time and failed then).

4. Calculate the cumulative hazard value for each failure as the sum of its hazard value and the hazard values of all preceding failure times. Cumulative hazard values can be larger than 100%. The sample cumulative hazard values provide a nonparametric estimate of the cumulative hazard function of the true distribution.

5. Choose the hazard paper of a theoretical distribution for time to failure.

5. Plot each failure time vertically against its corresponding cumulative hazard value or its horizontal axis of the hazard paper.

7. If the plot of the sample times to failure is reasonably straight on a hazard paper, one may conclude that the distribution adequately fits the data. If the data do not follow a reasonably straight line, then plot on hazard paper for some other theoretical distribution to see if a better fit can be obtained.


X →
n ≅ 110
100%
← SLOPE (MEASURED WITH RULER) =
⇒ B ≅ 1.9
10
9
8
7
6
5
4
3
2
100
90
80
70
60
50
40
30
20
10
1
2
3
4
5
6
7
8
9 10
20
30
40
50
60
70 80 90 100
200
3
K&E LOGARITHMIC 46 7400
3 X 3 CYCLES MADE IN U.S.A.
KEUFFEL & ESSER CO.

P_g II 71

1. A: OWNS CAR

B: OWNS HOUSE

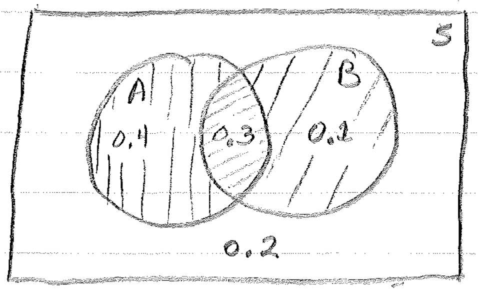
C: A ∪ B

$P[e_1] = P[A \cap B] = 0.3$

$P[e_2] = P[A \cap \bar{B}] = 0.4$

$P[e_3] = P[\bar{A} \cap B] = 0.1$

$P[e_4] = P[\bar{A} \cap \bar{B}] = 0.2$



$P[A] = 0.7$

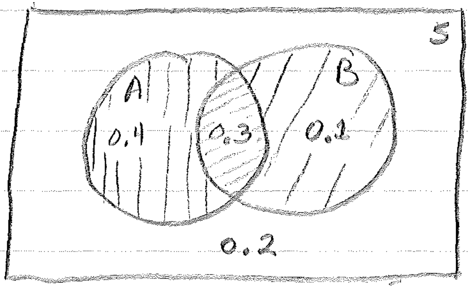
$P[B] = 0.4$

$P[C] = P[A \cup B] = 0.8$

Pg II 71

1. A: OWNS CAR
B: OWNS HOUSE
C: A ∪ B

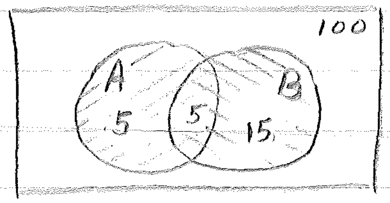
$P[e_1] = P[A \cap B] = 0.3$
$P[e_2] = P[A \cap \bar{B}] = 0.4$
$P[e_3] = P[\bar{A} \cap B] = 0.1$
$P[e_4] = P[\bar{A} \cap \bar{B}] = 0.2$



$P[A] = 0.7$
$P[B] = 0.4$
$P[C] = P[A \cup B] = 0.8$

Pg II 71

2.



$P[A] = P[A \text{ FAILS}] = 0.10$

$P[B] = P[B \text{ FAILS}] = 0.20$

$P[A \cap B] = P[A \text{ AND } B \text{ FAILS}] = 0.05$

a. FIND $P[A/B]$

SINCE $P[A \cap B] = P[B]\,P[A/B]$

$$\Rightarrow P[A/B] = P[A \cap B]/P[B]$$
$$= 0.05/0.20$$
$$= \frac{1}{4}$$ ✓

b. FIND $P[A/\bar{B}]$

$P[A/\bar{B}] = P[A \cap \bar{B}]/P[\bar{B}]$

FROM VENN DIAGRAM: $P[A \cap \bar{B}] = \frac{5}{100} = 0.05$

$$\Rightarrow P[A/\bar{B}] = 0.05/0.80$$
$$= \frac{1}{16}$$ ✓

c. FIND $P[B/A]$

SINCE $P[A \cap B] = P[A]\,P[B/A]$

$$\Rightarrow P[B/A] = P[A \cap B]/P[A]$$
$$= 0.05/0.10$$
$$= \frac{1}{2}$$ ✓

d. $P[B/\bar{A}] = P[\bar{A} \cap B]/P[\bar{A}]$

FROM VENN DIAGRAM: $P[\bar{A} \cap B] = \frac{10}{100} = 0.15$

$$\Rightarrow P[B/\bar{A}] = \frac{0.15}{0.90}$$
$$= \frac{1}{6}$$ ✓

(CONT.)

(CONT.)

✓e. $P[A] = 0.10$

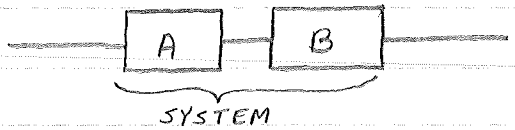
✓f. $P[B] = 0.20$

✓g. FROM (b) $\Rightarrow$ $P[A \cap \bar{B}] = 0.05$

✓h. NO $\{ P[A] \neq P[A/B]$ ETC. $\}$

Pg II 72

3.



$P_s = P[\text{SYSTEM WORKS}]$

$= P[\text{A WORKS } \underline{\text{AND}} \text{ B WORKS}]$

ASSUMING INDEPENDENCE (NESCESSARY REASONABLE ASSUMPTION)

$P_s = P[\text{A WORKS}]P[\text{B WORKS}]$

$= P_A P_B$

$= (0.9)(0.8)$

$= 0.72$

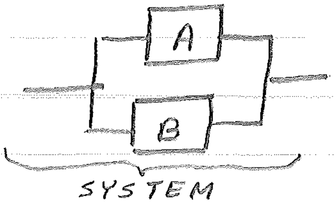
$\bar{P}_s = P[\text{SYSTEM WILL FAIL}]$

$= 1 - P_s$

$= 0.28$ ✓

Pg II 72

4.



P_s = P[SYSTEM WILL WORK]

= P[A WILL WORK OR B WILL WORK OR BOTH WILL WORK]

$= P[A \cup B] = P[A] + P[B] - P[A \cap B]$

FOR INDEPENDENT EVENTS

$$P_s = P[A \cup B] = P[A] + P[B] - P[A]P[B]$$

$$= P[A] + P[B]\{1 - P[B]\}$$

$$= 0.8 + 0.7\{0.2\}$$

$$= 0.8 + 0.14$$

$$= 0.94$$ ✓

Pg II 72

5. $P[A] = P[\text{ITEM SURVIVES}^*]$

a. FIND

$P_1 = P[\text{AT LEAST ONE OF THREE SURVIVES}]$

$= P[\text{ONE SURVIVES } \overset{\cup}{\underline{\text{OR}}} \text{ TWO SURVIVE } \overset{\cup}{\underline{\text{OR}}} \text{ THREE SURVIVE}]$

MUTUAL EXCLUSIVENESS DICTATES

$P_1 = P[\text{ONE SURVIVES}] + P\{\text{TWO SURVIVE}] + P\{\text{THREE SURVIVE}]$

ASSUMING INDEPENDENCE:

$$P[\text{TWO SURVIVE}] = P[A_1]P[A_2]P[\bar{A}_3] + P[A_1]P[\bar{A}_2]P[A_3] + P[\bar{A}_1]P[A_2]P[A_3]$$

$$= 3[0.8]^2 0.2$$

$$= 0.384$$

$$P[\text{THREE SURVIVE}] = P[A_1]P[A_2]P[A_3]$$

$$= (0.8)^3$$

$$= 0.512$$

$$P[\text{ONE SURVIVES}] = 3\,P[\bar{A}_1]P[\bar{A}_2]P[A_3]$$

$$= 3(0.2)^2(0.8) = 0.096$$

$\Rightarrow P_1 = 0.384 + 0.512 + 0.096 = 0.992$ ✓

b. $P_1 = P[\text{AT LEAST TWO ITEMS SURVIVE}]$

$= P[\text{TWO ITEMS SURVIVE } \underline{\text{OR}} \text{ THREE ITEMS SURVIVE}]$

$= P[\text{TWO SURVIVE}] + P[\text{THREE SURVIVE}]$

$= 0.384 + 0.512$

$= 0.896$ ✓

---

* FOR 10 HRS

Pg. II-30

1. FIND MEAN OF

1, 9, 8, 15, 4, 8

$$\mu = \frac{1}{6}\sum_{i=1}^{6} x_i = \frac{1}{6}[45] = 7.5 \checkmark$$

2. FIND MEAN OF

5, 6, 8, 7, 106

$$\mu = \frac{1}{5}\sum_{i=1}^{5} x_i = \frac{1}{5}(132) = 26.4 \checkmark$$

Pg. II-36

2. COMPUTE σ FOR
5, 6, 8, 7, 106

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2 = \frac{1}{n^2}\left[n\sum x_i^2 - (\sum x_i)^2\right]$$

$\sum x_i = 132 \Rightarrow (\sum x_i)^2 = 17424$
$\sum x_i^2 = 11410$
$n = 5$ ✓

$$\Rightarrow \sigma^2 = \frac{11410}{5} - \frac{17424}{25}$$

$= 2282 - 696.96$
$= 1585.04$

$\Rightarrow \sigma = 39.81256$ ✓

Pg. II-43

1. a. FOR $P = 0.2$, $P[X = 4] \simeq 0.09$ ✓
   b. FOR $P = 0.5$, $P[X = 4] \simeq 0.21$ ✓
   c. FOR $P = 0.7$, $P[X = 4] \simeq 0.04$ ✓

2. $$P[X = x] = \binom{n}{x} p^x q^{n-x}$$
   $$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

   FOR b IN # 1:

   $n = 10$, $x = 4$, $p = q = \frac{1}{2}$

   $$\Rightarrow P[X = 4] = \frac{10!}{4!\,6!} \left(\frac{1}{2}\right)^{10}$$

   $$= \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \left(\frac{1}{2}\right)^{10}$$

   $$= \frac{210}{2^{10}}$$

   $2^5 = 32 \Rightarrow 2^{10} = 1024$

   AND $P[X = 4] = \frac{210}{1024} = \frac{105}{512}$

   $= 0.2051$ ✓

Pg II 3.

$p = \frac{1}{6}$

$q = \frac{5}{6}$

$$\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + \binom{5}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + \binom{5}{5}\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0$$

$$\frac{5 \cdot 4}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + 5\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + \left(\frac{1}{6}\right)^5$$

$$= 0.032 + 0.00321 + .000128$$

$$= 0.03549 \checkmark$$

Pg II - 43

4. $q = 0.1$; $p = 0.9$

$n = 5$

$\therefore P[X = x] = \binom{n}{x} p^x q^{n-x}$

a. FIND $P[X = 0]$

$$= \binom{5}{0} (0.9)^0 (0.1)^5$$
$$= 10^{-5}$$

b. FIND $P[X = 1]$

$$= \binom{5}{1} (0.9)^1 (0.1)^4$$
$$= 5 \times 0.9 \times 10^{-4} = 4.5 \times 10^{-4}$$

c. FIND $P[X > 1]$

$$= 1 - \{P[X = 0] + P[X = 1]\}$$
$$= 1 - \{45 \times 10^{-5} + 10^{-5}\}$$
$$= 1 - 46 \times 10^{-5}$$
$$= 1 - 0.00046$$
$$= 0.99954 \checkmark$$

5. $p = 0.73 \quad q = 0.27$

$n = 7$

$P[3 \text{ FAIL}] = \binom{7}{3} (0.27)^3 (0.73)^4$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} (0.27)^3 (0.73)^4$

$= 0.195637$ ✓

Pg II-72

6. $\frac{1}{\lambda} = 200$ HRS

$T = 7$ HRS

FIND $R(7)$, $h(5)$

a. FOR EXPONENTIAL:

$$R(x) = e^{-\lambda t}\mu(t)$$

$$\Rightarrow R(7) = e^{-\frac{7}{200}} = 0.9656$$ ✓

b. FOR EXPONENTIAL

$$h(x) = \lambda \Rightarrow h(5) = \frac{1}{200} \ \frac{\text{FAILURES}}{\text{HR}}$$

$$= 5 \times 10^{-3} \ \frac{\text{FAILURES}}{\text{HR}}$$ ✓

7. TIME USED = 80 HRS

SINCE THE HAZARD RATE IS CONSTANT, IT MATTERS NOT HOW LONG THE ITEM IS USED. THE RELIABILITY FOR THE PLANNED USE IS THUS:

$$R(10) = e^{-\frac{10}{200}} = 0.951229$$ ✓

Pg II-72

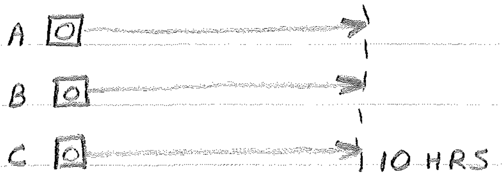
8. GIVEN

$\frac{1}{\lambda} = 100$ HRS

$N = 3$ ITEMS $X = 10$ HRS

FIND:

a. P[NO FAILURES]



A = P[A WORKS]

B = " B "

C = " " "

$P[\text{NO FAILURES}] = P[A \cap B \cap C]$

ASSUMING INDEPENDENCE:

$P[\text{NO FAILURES}] = P[A]P[B]P[C]$

WE ALSO KNOW THAT $P[A] = P[B] = P[C]$

THUS $P[\text{NO FAILURES}] = \{P[A]\}^3$

$P[A] = P[X > 10 \text{ HRS}]$

GENERALLY, FOR THE EXPONENTIAL

$P[X \geq x] = R(x) = e^{-\lambda x}$

THUS

$$P[X > 10] = R(10) = e^{-\left(\frac{10}{100}\right)} = e^{-0.1}$$

AND

$$P[\text{NO FAILURES}] = (e^{-0.1})^3 = e^{-0.3}$$

$$= 0.7408182 \checkmark$$

(CONT) Pg II-72

8b. FIND $P[X=1]$

$$P[X=1] = P[(\bar{A}\cap B\cap C)\cup(A\cap\bar{B}\cap C)\cup(A\cap B\cap\bar{C})]$$

THESE EVENTS ARE MUTUALLY EXCLUSIVE. THUS

$$P[X=1] = P[\bar{A}\cap B\cap C] + P[A\cap\bar{B}\cap C] + P[A\cap B\cap\bar{C}]$$

SINCE THE ITEMS ARE IDENTICAL

$$P[\bar{A}\cap B\cap C] = P[A\cap\bar{B}\cap C] = P[A\cap B\cap\bar{C}]$$

AND

$$P[X=1] = 3P[\bar{A}\cap B\cap C]$$

THE EVENTS PROBABILITIES ARE IDENTICAL, SO THAT

$$P[X=1] = 3P[\bar{A}]P[B]P[C]$$

AGAIN, BECAUSE THE UNITS ARE IDENTICAL

$$P[B] = P[C] = P[A]$$

THUS

$$P[X=1] = 3\{P[A]\}^2\{1-P(A)\}$$

NOW

$$P[A] = P[X>x] = R(x)$$

FOR EXPONENTIAL

$$R(x) = e^{-\lambda x}$$
$$= e^{-\frac{10}{100}} = e^{-0.1}$$

THUS

$$P[X=1] = 3e^{-0.2}[1-e^{-0.1}]$$
$$= 0.2337376 \checkmark$$

Pg II-72

ALTERNATE SOLUTION EMPLOYING BINOMIAL

8. a. $P[\underline{X}=0] = \binom{3}{0}[1-R(10)]^0[R(10)]^3$

$= [R(10)]^3$

b. $P[\underline{X}=1] = \binom{3}{1}[1-R(10)]^1[R(10)]^2$

$= 3[1-R(10)]^1[R(10)]^2$

Pg II-72

9. GIVEN

$\beta = 2$

$\eta = 10$

FIND

a. $R(1)$

FOR A WEIBULL:

$$R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$$

THUS

$$R(1) = e^{-\left(\frac{1}{10}\right)^{2}} = e^{-\frac{1}{100}} = 0.99005$$ ✓

b. $h(5)$

FOR ANY DISTRIBUTION

$$h(x) \triangleq f(x)/R(x)$$

FOR THE WEIBULL

$$h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

THUS

$$h(5) = \frac{2}{10}\left(\frac{5}{10}\right)^{2-1} = 0.100$$ ✓

Pg. II-73

10. GIVEN

$\beta = 2$

$\eta = 10$ HRS

THE ITEM HAS BEEN USED FOR 2 HRS FIND THE RELIABILITY OF THE ITEM FOR AN ADDITIONAL 1 HR MISSION, THAT IS, FIND

$$P[X > 3 / X > 2] = \frac{R(3)}{R(2)}$$

FOR THE WEIBULL:

$$R(x) = e^{-(\frac{x}{\eta})^{\beta}} = e^{-(\frac{x}{10})^{2}}$$

$$\therefore R(3) = e^{-(\frac{3}{10})^{2}} = 0.913931$$

$$R(2) = e^{-(\frac{2}{10})^{2}} = 0.907894$$

$$\therefore P[X > 3 / X > 2] = 0.9512294$$ ✓

Pg II - 73

11. GIVEN

$\beta = 2$ $\quad$ $x = 20$ HRS

$\eta = 100$ HRS

4 ITEMS

FIND: $P[S] = P[\underline{X} = 2$ FAILURES IN 10 HRS$]$

$$P[S] = P[(A \cap B \cap \bar{C} \cap \bar{D}) \cup (A \cap \bar{B} \cap C \cap \bar{D}) \cup (\bar{A} \cap B \cap C \cap \bar{D})$$
$$\cup (A \cap \bar{B} \cap \bar{C} \cap D) \cup (\bar{A} \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap \bar{B} \cap C \cap D)]$$

DUE TO THE EQUALLY LIKELY MUTUALLY EXCLUSIVE EVENTS.

$$P[S] \overset{*}{=} 6\, P[A \cap B \cap \bar{C} \cap D]$$

DUE TO INDEPENDENCE

$$P[A] = P[B] = P[C] = P[D]$$

$$\Rightarrow P[S] = 6\{P[A]\}^2\{1 - P[A]\}^2 \Leftarrow$$ CAN GET STRAIGHT FROM BINOMIAL DISTRIBUTION

NOW: $P(A) = P[\underline{X} \geq x] = R(x)$

FOR THE WEIBULL:

$$R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$$

THUS: $P(A) = R(20) = e^{-\left(\frac{20}{100}\right)^2} = e^{-0.04}$

$$\Rightarrow P[S] = 6\,[e^{-0.04}]^2[1 - e^{-0.04}]^2$$
$$= 6\, e^{-0.08}[1 - e^{-0.04}]^2$$
$$= 0.0085155715 \checkmark$$

*NOTE: $6 = \binom{4}{2}$

Pg II-73

12. GIVEN

$\mu = 50$ HRS ; $\sigma = 10$ HRS

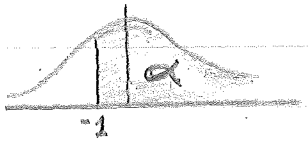
a. FIND $R(40)$

GENERALLY, FOR NORMAL DISTRIBUTION

$$R(x) = P[X > x]$$
$$= P\left[z > \frac{x-\mu}{\sigma}\right]$$

THUS

$$R(40) = P\left[z > \frac{40-50}{10}\right] = P[z > -1]$$
$$= 1 - P[z > 1]$$
$$= 1 - 0.15866$$



$\Rightarrow R(40) = 0.84134$ ✓

b. FIND $h(40)$

GENERALLY:
$$h(x) = \frac{f(x)}{R(x)}$$

NOW
$$f(z) = f\left[\frac{x-\mu}{\sigma}\right] = f[-1] = f[1]$$
$$= 0.2420$$

$$f(x) = \frac{f(z)}{\sigma} = 0.02420$$

$$\Rightarrow h(x) = \frac{0.02420}{0.84134} = 0.02876364$$ ✓

Pg II · 73

13. GIVEN

$\mu = 50$ HRS ; $\sigma = 10$ HRS

FIND

$$P[\underline{X} > 55 / \underline{X} > 45] = \frac{R(55)}{R(45)}$$

$$R(x) = P[\underline{X} > x] = P[z > \frac{x-\mu}{\sigma}]$$

$$\therefore R(55) = P[z > \frac{55-50}{10}] = P[z > 0.5]$$

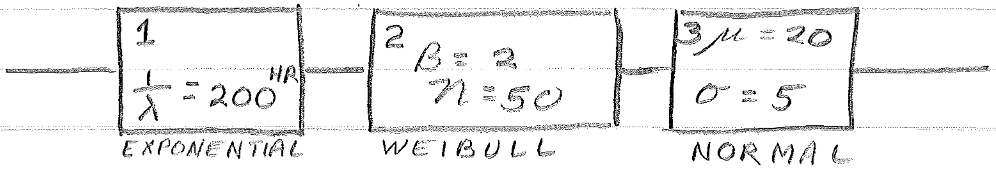
$$R(45] = P[z > \frac{45-50}{10}] = P[z > {}^{-}0.5]$$
$$= 1 - P[z > 0.5]$$

$$P[z > 0.5] = 0.30854$$

$$\Rightarrow P[\underline{X} > 55 / \underline{X} > 45] = \frac{0.30854}{1-0.30854}$$
$$= 0.4462153 \checkmark$$

Pg III - 46

1.



$$R_S(10) = \prod_{i=1}^{3} R_i(10)$$

FOR BLOCK 1

$$R_1(x) = e^{-\lambda x} \Rightarrow R(10) = e^{-\frac{10}{200}} = 0.9512294 \checkmark$$

FOR BLOCK 2

$$R_2(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}} \Rightarrow R(10) = e^{-\left(\frac{10}{50}\right)^2} = 0.9607894 \checkmark$$

FOR BLOCK 3

$$R_3(x) = P[X > x]$$

$$= P\left[z > \frac{x-\mu}{\sigma}\right]$$

$$= P\left[z > \frac{10-20}{5} = -2\right]$$

$$= 1 - P[z > 2]$$

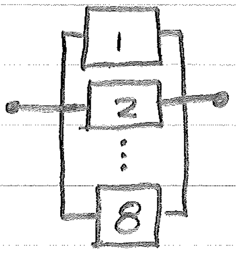
$$= 1 - 0.02275$$

$$= .97725 \checkmark$$

$$\Rightarrow R_S(10) = 0.893139 \checkmark$$

Pg III - 46

2.



$R_i = 0.8 \Rightarrow Q_i = 0.2$

SHOULD HAVE BEEN WORKED FOR N=5

$$Q_s = \prod_{i=1}^{n} Q_i$$

$$= Q_i^{\,n}$$

$$= (0.2)^8 = 2.56 \times 10^{-6}$$

$$= ,00000256$$

$$R_s = 1 - Q_s = .99999744$$

3. $Q_s = \prod_{i=1}^{n} Q_i$

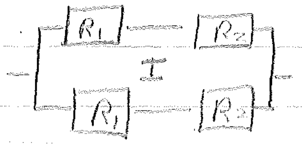
$$= [1-0.8]^5 [1-0.6]^2$$

$$= (0.2)^5 (0.4)^2 = 5.12 \times 10^{-5}$$

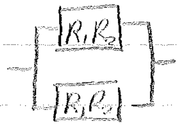
$$= 0,0000512$$

$$R_s = 1 - Q_s = 0,9999488$$

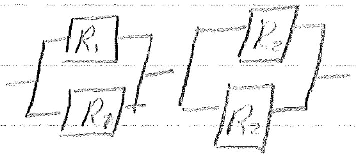
Pg III - 46

4. 

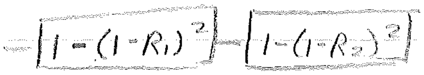
$\equiv$



$\Rightarrow R_I = 1-(1-R_1R_2)^2$



$=$



$\Rightarrow R_{II} = [1-(1-R_1)^2]$
$\times [1-(1-R_2)^2]$

R_I	R_{II}
$1-(1-R_1R_2)^2$	$[1-(1-R_1)^2][1-(1-R_2)^2]$
$1-[1-2R_1R_2+R_1^2R_2^2]$	$[1-(1-2R_1+R_1^2)][1-(1-2R_2+R_2^2)$
$2R_1R_2-R_1^2R_2^2$	$(2R_1-R_1^2)(2R_2-R_2^2)$
	$4R_1R_2-2R_1R_2^2-2R_2R_1^2+R_1^2R$

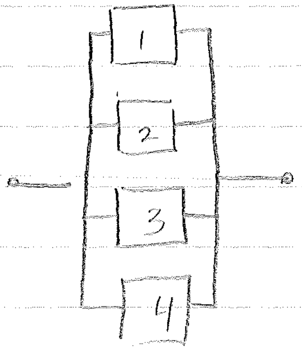
COMPARE

R_I		R_{II}
$2R_1R_2-R_1^2R_2^2$	:	$4R_1R_2-2R_1R_2(R_2+R_1)+R_1^2R_2^2$
$-R_1^2R_2^2$	;	$2R_1R_2-2R_1R_2(R_2+R_1)+R_1^2R_2^2$
0	;	$-2R_1R_2(R_1+R_2-1\quad)+2R_1^2R_2^2$
	;	$-2R_1R_2(R_1+R_2-1-R_1R_2)$

II IS BEST

Pg III-47

5.



$R_i = 0.8$

$R_s = P[X \le 2] = P[X \ge 2 \text{ succ}]$

$p = R_i = 0.8 \quad q = 0.2$

$$P[X \ge 2 \text{ suc}] = \sum_{x=2}^{4} \binom{4}{x} (0.8)^x (0.2)^{4-x}$$

$$= \binom{4}{2}(.8)^2(.2)^2 + \binom{4}{3}(0.8)^3(0.2)^1$$

$$+ \binom{4}{4}(0.8)^4(0.2)^0$$

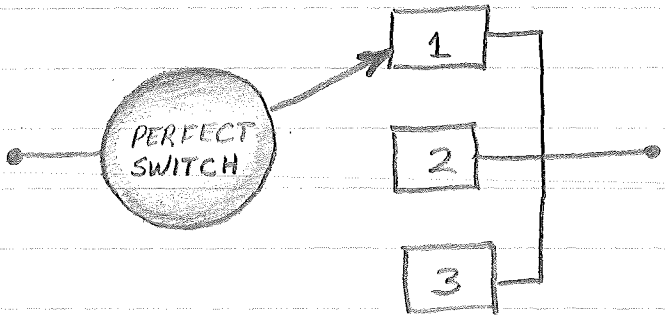
$$= 6(0.64)(0.04) + 4(0.512)(0.2) + (0.4096)$$

$$= 0.1536 + 2(0.4096)$$

$$= 0.9728$$

Pg III - 47

6.



EACH ELEMENT IS EXPONENTIALLY DISTRIBUTED WITH $\frac{1}{\lambda} = 200$ HRS.

FIND $R(100)$

FOR SUCH A SYSTEM:

$$R(x) = \sum_{K=0}^{n} \frac{(\lambda x)^K e^{-\lambda x}}{K!}$$

WHERE $n+1$ = # OF SUBSYSTEMS

$$\therefore R(100) = \sum_{K=0}^{2} \frac{\left(\frac{100}{200}\right)^K e^{-\frac{100}{200}}}{K!}$$

$$= \sum_{K=0}^{2} \frac{(0.5)^K e^{-0.5}}{K!}$$

$$= \frac{(0.5)^0 e^{-0.5}}{0!} + \frac{(0.5)^1 e^{-0.5}}{1!} + \frac{(0.5)^2 e^{-0.5}}{2!}$$

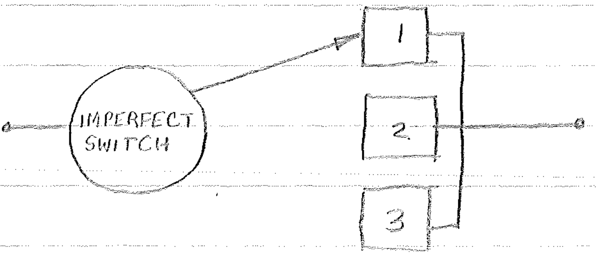
$$= [1 + 0.5 + 0.125] e^{-0.5}$$

$$= 1.625 \times e^{-0.5}$$

$$= 0.9856123$$

Pg. III-48

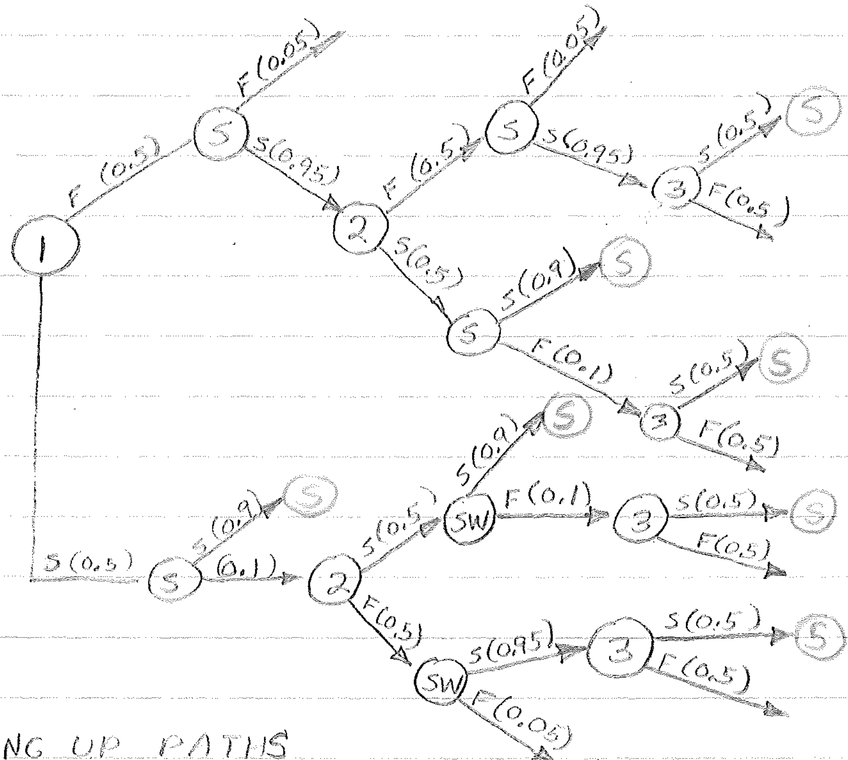
8.



$p_1 = p_2 = p_3 = q_1 = q_2 = q_3 = 0.5$

$q_w' = P[\text{PREMATURE SWITCHING}] = 0.1$; $p_w' = 0.9$

$q_w = P[\text{NOT SWITCHING}] = 0.05$; $p_w = 0.95$



ADDING UP PATHS

$$R_s = (0.5)^3(0.95)^2 + [(0.5)^2(0.95)][0.9 + 0.05] + (0.5)(0.9) + (0.5)^2(0.1)[(0.9) + (0.05)] + (0.5)^3(0.095)$$

$$= 0.83$$

Pg III-49

10, $R^* = 0.95$

$n = 4$

$x = 100$ HRS

$\lambda_1 = 10 \times 10^{-4}$; $\lambda_2 = 4 \times 10^{-4}$; $\lambda^3 = 10^{-4}$; $\lambda_4 = 3 \times 10^{-4}$

$\sum_{i=1}^{4} \lambda_i = 18 \times 10^{-4}$

$w_i = \lambda_i / \Sigma \lambda_i$

$W_1 = \frac{10 \times 10^{-4}}{18 \times 10^{-4}} = 0.55556$

$W_2 = \frac{4 \times 10^{-4}}{18 \times 10^{-4}} = 0.22222$

$W_3 = \frac{10^{-4}}{18 \times 10^{-4}} \; 0.055556$

$W_4 = \frac{3 \times 10^{-4}}{18 \times 10^{-4}} = 0.166667$

$R^* = e^{-\lambda^* x} = e^{-\lambda^* 100} \Rightarrow \lambda^* = \frac{-1}{100} \ln R^* = 5.129 \times 10$

$\lambda_i^* \leq w_i \lambda^*$

$\lambda_1^* = (0.55556)(5.129 \times 10^{-4}) = 2.849 \times 10^{-4}$

$\lambda_2^* = (0.22222)(5.129 \times 10^{-4}) = 1.1399 \times 10^{-4}$

$\lambda_3^* = (0.05556)(5.129 \times 10^{-4}) = 2.8496 \times 10^{-5}$

$\lambda_4^* = (0.166667)(5.129 \times 10^{-4}) = (8.54882 \times 10^{-5}$

Now

$R_i^* = e^{-\lambda^* x}$

$\therefore R_1^* = 0.971906$

$R_2^* = 0.988666$

$R_3^* = 0.971544$

$R_4^* = 0.991488$

Pg IV-237

1. $T=50$

$n=10$

$r=4 \Rightarrow d=6$

a. $\hat{R}(50) = \frac{d}{n} = 0.60$

b. FROM TABLE K_1

90% CONFIDENT THAT $0.304 \leq R(50) \leq 0.850$

c. FROM TABLE K_2

95% CONFIDENT THAT $0.850 \leq R(50)$

Pg IV-237

2. 2, 6, 8, 15, 25, 26, 29, 40, 42, 50

FROM TABLE H_1, $n=10$

$r=4 \Rightarrow \tilde{R}(15) = 1 - 0.3557 = 0.6443$

$r=5 \Rightarrow \tilde{R}(25) = 1 - 0.4519 = 0.5481$

SINCE 20 IS MEAN OF 15 AND 25

$\tilde{R}(20) = \frac{0.5481 + 0.6443}{2} = \frac{1.1924}{2} = 0.5962$

3.a. FOR $R(25)$, $r = 5$

FROM TABLE H_2: $\tilde{R}_u(25) = 1 - \tilde{F}_l(25)$

$= 1 - 0.2224$

$= 0.7776$

FROM TABLE H_3: $R_\ell(25) = 1 - \tilde{F}_u(25)$

$= 1 - 0.6965$

$= 0.3035$

$\therefore$ 90% CONFIDENT $0.3035 \leq R(25) \leq 0.7776$

b. FOR $R(40)$, $r = 8$

FROM TABLE H_2

$\tilde{R}_\ell(40) = 1 - 0.9127$

$= 0.0873$

$\therefore$ 95% CONFIDENT $R(25) \geq 0.0873$

Pg IV-238

4.

i	X_i	EXPONENTIAL $\frac{n+1}{n+1-i} = \frac{11}{11-i}$	NORMAL WEIBULL $\frac{100i}{n+1} = \frac{100i}{11}$
1	12.86	$\frac{11}{10} = 1.100$	9.0909
2	16.89	$\frac{11}{9} = 1.222$	18.182
3	18.79	$\frac{11}{8} = 1.375$	27.273
4	21.00	$\frac{11}{7} = 1.571$	36.364
5	22.04	$\frac{11}{6} = 1.833$	45.455
6	23.50	$\frac{11}{5} = 2.200$	54.545
7	25.06	$\frac{11}{4} = 2.750$	63.636
8	27.30	$\frac{11}{3} = 3.667$	72.727
9	27.75	$\frac{11}{2} = 5.500$	81.818
10	37.54	$\frac{11}{1} = 11.000$	90.909

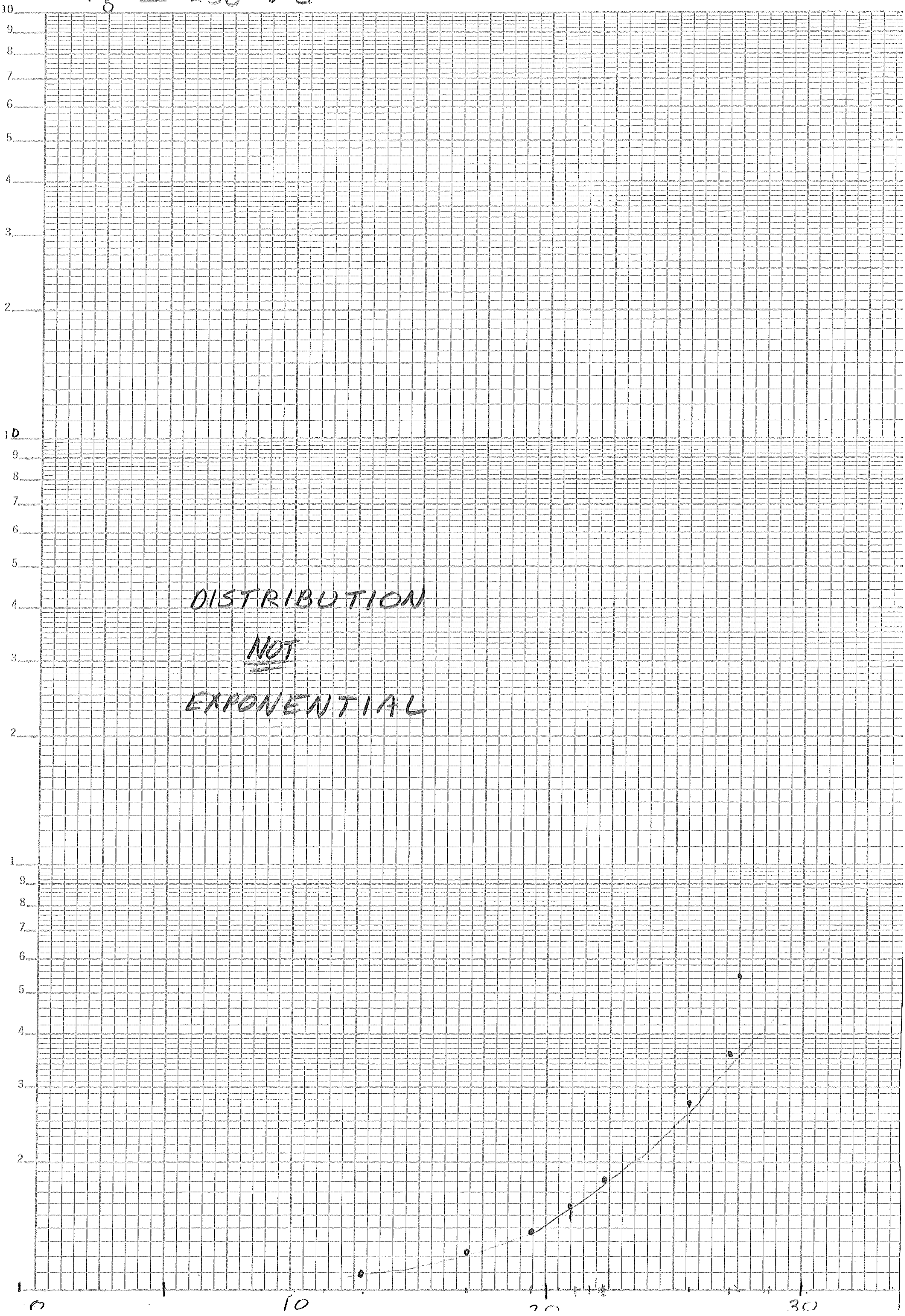
COMPUTED:

$\bar{X} = 21.6878$

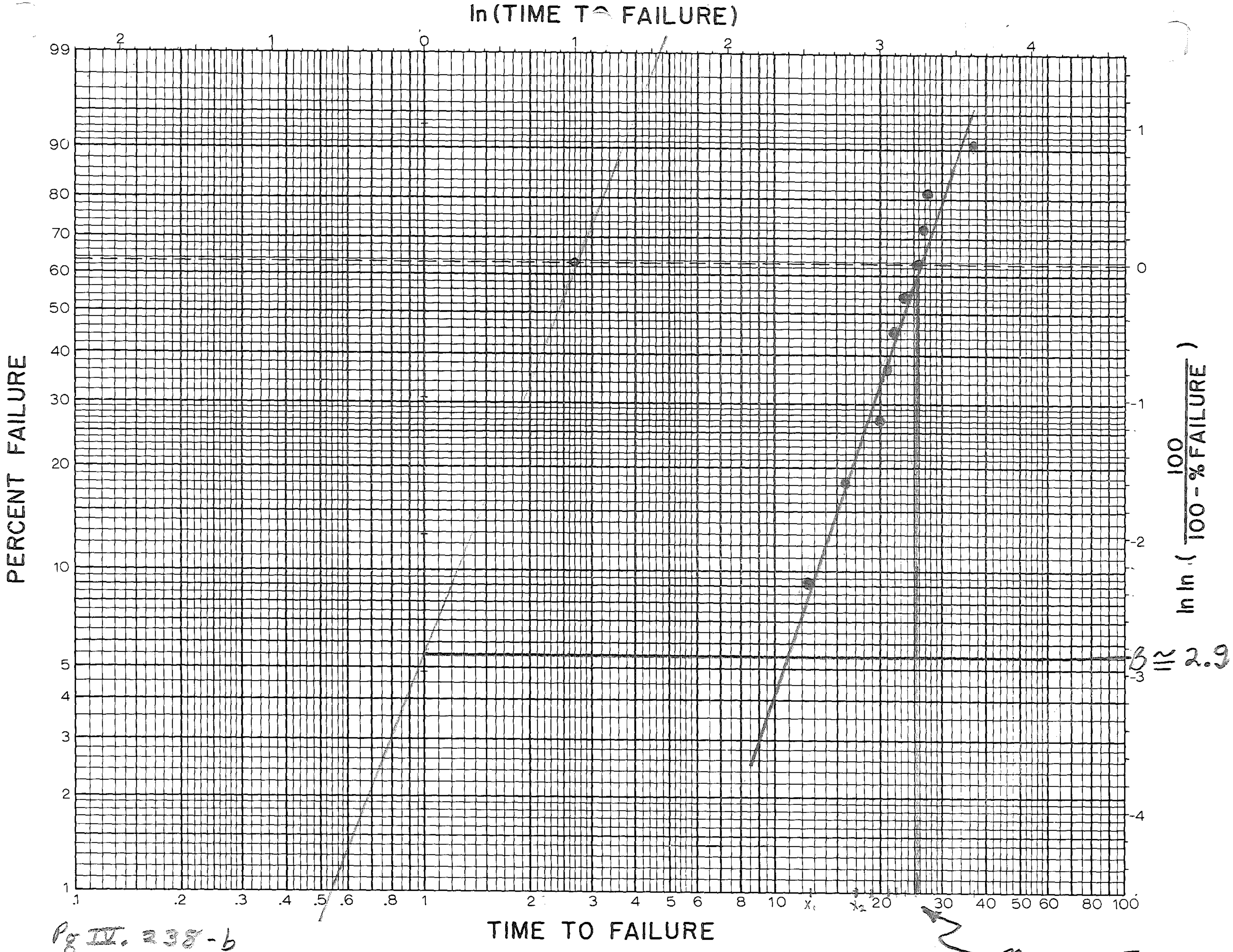
$\Sigma X^2 = 442.61$

$s = 4.909763$

Pg IV-238 → a



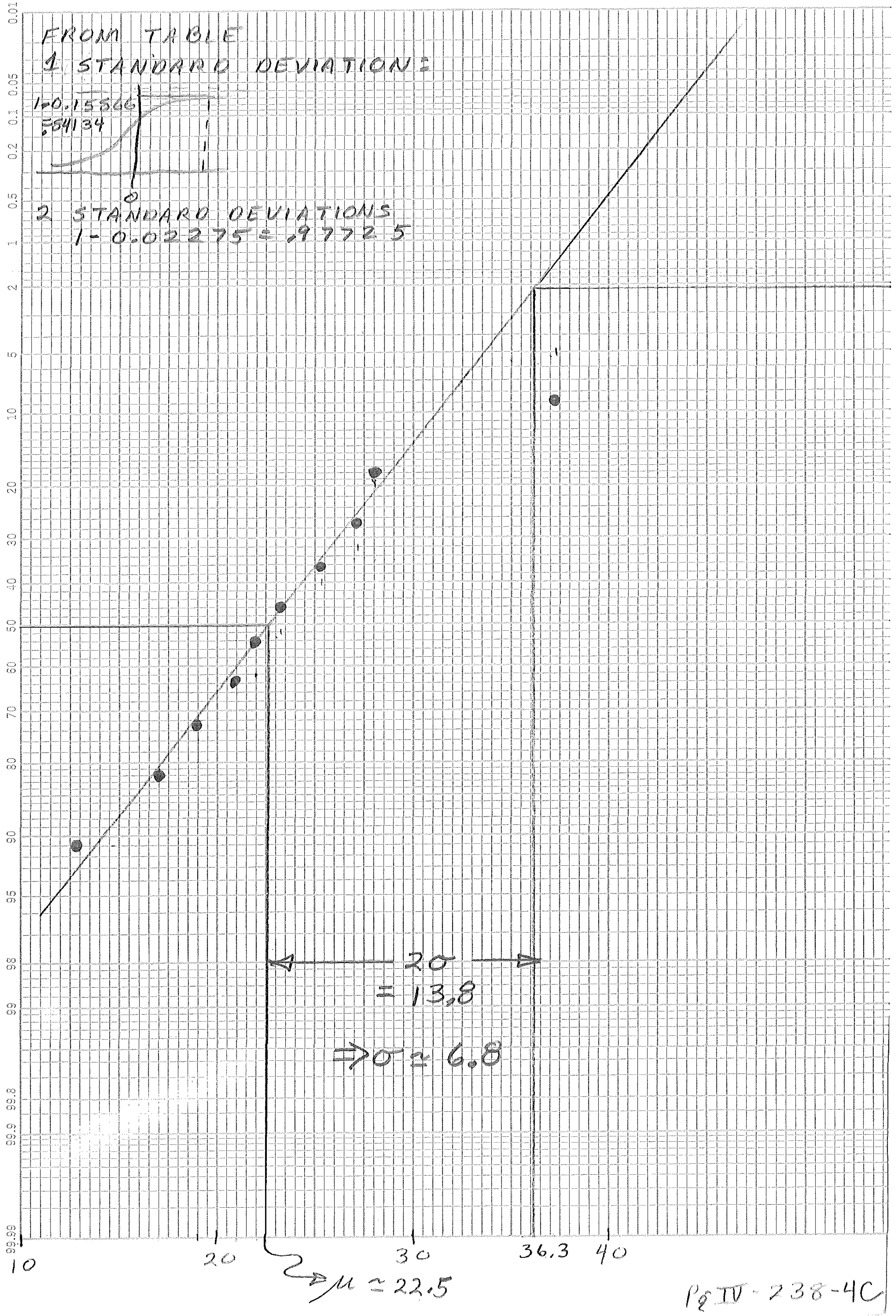
DISTRIBUTION
NOT
EXPONENTIAL



$\beta \cong 2.9$

$\eta = 25.5$

Pg IV. 238-b
WEIBULL



FROM TABLE
1 STANDARD DEVIATION:
1-0.15866
=.84134

2 STANDARD DEVIATIONS
1-0.02275=.97725

2σ
=13.8

⇒σ≈6.8

μ≈22.5

Pg IV-239

6.

i	x_i	$\hat{F}(x_i)$
1	2	1/7
2	27	2/7
3	56	3/7
4	64	4/7
5	80	5/7
6	117	6/7
7	122	1

$\alpha = 0.05$ (4)

$\bar{X} = 66.85714286$

$S = 44.06218549$

a. EXPONENTIAL : $\hat{\theta} = \bar{X}$

$F(x_i) = 1 - e^{-x_i/\bar{x}}$ $\qquad d_i = |F(x_i) - \hat{F}(x_i)|$

$= |\hat{F}(x_i) - F(x_i)|$

1. $F(2) = 1 - e^{-2/\bar{x}}$

$d_1 = |\frac{1}{7} - 1 + e^{-2/\bar{x}}| = 0.113386$

2. $F(27) = 1 - e^{-27/\bar{x}}$

$d_2 = |\frac{2}{7} - 1 + e^{-27/\bar{x}}| = 0.046539$

3. $F(56) = 1 - e^{-56/\bar{x}}$

$d_3 = |\frac{3}{7} - 1 + e^{-56/\bar{x}}| = 0.138684$

4. $F(64) = 1 - e^{-64/\bar{x}}$

$d_4 = |\frac{4}{7} - 1 + e^{-64/\bar{x}}| = 0.0446299$

5. $F(80) = 1 - e^{-80/\bar{x}}$

$d_5 = |\frac{5}{7} - 1 + e^{-80/\bar{x}}| = 0.0165114$

6. $F(117) = 1 - e^{-117/\bar{x}}$

$d_6 = |\frac{6}{7} - 1 + e^{-117/\bar{x}}| = 0.0309168$

7. $F(122) = 1 - e^{-122/\bar{x}}$

$d_7 = |e^{-122/\bar{x}}| = 0.1612521 \Longleftarrow$ MAXIMUM

REJECT IF $d = 0.161 > d_{\alpha;n}$

$d_{\alpha;n} = d_{0.05;7} = 0.486$

$\therefore$ CANNOT REJECT

Pg IV·239

7. 25, 30, 41, 43 $\Rightarrow n = 4$

a. $\bar{x} = \frac{1}{n}\sum x_i = 34.75 = \hat{\mu}$

$s = \left[\frac{1}{n}\sum(x_i - \bar{x})^2\right]^{1/2} = 8.65544 = \hat{\sigma}$

b. $\alpha = 0.10$

$\Rightarrow$ FOR MEAN

$$\bar{x} - t_{\alpha/2;n-1}\left(\frac{s}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + t_{\alpha/2;n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$34.75 - t_{0.05,3}(4.3277) \leq \mu \leq 34.75 + t_{0.05;3}(4.3277)$$

$$t_{0.05,3} = 2.353$$

$$\Rightarrow 24.57 \leq \mu \leq 44.93$$

$\rightarrow$ FOR STANDARD DEVIATION

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2};n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2};n-1}}$$

$$(n-1)s^2 = 225$$

$$\chi^2_{0.05;3} = 7.815$$

$$\chi^2_{0.95;3} = 0.352$$

$$\Rightarrow 28.791 < \sigma^2 < 639.2$$

$$5.366 < \sigma < 25.28$$

Pg $\underline{IV}$ - 239

8. (SEE PROB. 7)

a. $\hat{R}(30) = P[X > 30]$

$= P\left[Z = \frac{x-\mu}{s} \geq \frac{30 - 34.75}{8.655} = {}^{-}0.5488\right]$

$= 1 - P[Z > 0.5488]$

$= 1 - 0.29116$

$= 0.70884$

b. $\alpha = 0.20 \Rightarrow 1 - \alpha = 0.80$

$\hat{R}(x) \pm Z_{\alpha/2}\sqrt{V[\hat{R}(x)]}$

$V[\hat{R}(x)] = \frac{1}{n}\left[f\left(Z = \frac{x-\bar{x}}{s}\right)^2\right]\left[1 + \frac{1}{2}\left(\frac{x-\bar{x}}{s}\right)^2\right]$

$= \frac{1}{4}\left[f[0.549]^2\right] \times \left[1 + \frac{1}{2}(0.549)^2\right]$

$= \frac{1}{4}(0.3429)^2\left[1 + \frac{1}{2}(0.549)^2\right]$

$= 0.0339$

$\sqrt{V[\hat{R}(x)]} = 0.18392$

$Z_{\alpha/2} = Z_{0.10} = 1.28$ 

$\Rightarrow 0.70884 \pm (1.28)(0.184)$

0.70884 ± 0.23552

$\therefore$ 80% CONFIDENT THAT

$0.473 \leq R(30) \leq 0.944$

c. LOWER SIDED CONF. INTERVAL 95%

SAME AS ABOVE, BUT $Z_{\alpha/2} = Z_{0.05} = 1.64$

$0.709 - (1.64)(0.184) = 0.407$

$\therefore$ 95% CONFIDENT THAT

$R(30) > 0.407$

Pg IV - 239-40

9. $n = 5 \quad r = 3 \quad d = 2$

$t_i \Rightarrow 0.2,\ 0.7,\ 0.9$

a. $$\hat{\Theta} = \frac{X_t}{r} = \frac{d\,X_r + \sum_{i=1}^{r} X_i}{r}$$
$$= \frac{(2)(0.9) + [(0.2) + (0.7) + (0.9)]}{3}$$
$$= \frac{1.8 + 1.8}{3} = \frac{3.6}{3} = 1.2$$

BEST RELIABILITY ESTIMATE IS

$$\hat{R}(x) = e^{-x/\hat{\theta}}$$
$$\hat{R}(2) = e^{-(2/1.2)} = 0.18888$$

b. $$\frac{2r\hat{\theta}}{\chi^2_{\alpha/2;2r}} \leq \theta \leq \frac{2r\hat{\theta}}{\chi^2_{1-\frac{\alpha}{2};2r}} \quad ; \alpha = 0.05$$

$$\chi^2_{\alpha/2;2r} = \chi^2_{0.025;6} = 14.449$$
$$\chi^2_{1-\frac{\alpha}{2};2r} = \chi^2_{0.975;6} = 1.237$$

$$\frac{2(3)(1.2)}{14.449} \leq \theta \leq \frac{(2)(3)(1.2)}{1.237}$$

$0.4983 \leq \theta \leq 5.8205$ ⇐ 95% CONF

THUS

$$e^{\frac{-x}{5.8205}} \geq R(x) \geq e^{\frac{-x}{0.4983}}$$
$$e^{\frac{-2}{5.8305}} \geq R(2) \geq e^{\frac{-2}{0.4983}}$$

$0.7096 \geq R(2) \geq 0.01807$ ⇐ 95% CONF

c. $$\chi^2_{\alpha;2r} = \chi^2_{0.05;6} = 12.592$$
$$\chi^2_{1-\alpha;2r} = \chi^2_{0.95;6} = 1.635$$

FOR MEAN $\frac{(2)(3)(1.2)}{12.592} \leq \theta$

$\Rightarrow 0.5718 \leq \theta$ ⇐ 95% CONF.

FOR RELIABILITY $e^{-2/.5718} = 0.03267$

$\Rightarrow 0.03267 \leq R(2)$ ⇐ 95% CONF

Pp. 240-1

12. $n = 5$

x_i : 8, 40, 50, 60, 107

DISTRIBUTION IS WEIBULL

a. FINDING $\hat{\beta}$ & $\hat{\eta}$ VIA MATCHING MOMENTS:

$\bar{x} = 53 \Rightarrow 2809$

$S = 3.594 \Rightarrow S^2 = 1292$

$\frac{S^2}{\bar{x}^2} = \frac{1292}{2809} = 0.45995$

FROM MAGICAL b GRAPH

$b = 0.66$

$b = 1/\hat{\beta} \Rightarrow \hat{\beta} = 1.52$

$$\eta = \frac{\bar{x}}{\Gamma[1+b]} = \frac{\bar{x}}{\Gamma[1.66]} = \frac{53}{0.902} = 58.8$$

13. a. FOR WEIBULL

$$R(x) = e^{-(\frac{x}{\eta})^{\beta}}$$

THUS

$$\hat{R}(x) = e^{-(\frac{x}{\hat{\eta}})^{\hat{\beta}}}$$

AND

$$\hat{R}(1) = e^{-(\frac{1}{58.8})^{1.52}} = 0.99796$$

Pp. 240-1

12. $n=5$

x_i : 8, 40, 50, 60, 107

DISTRIBUTION IS WEIBULL

a. FINDING $\hat{\beta}$ & $\hat{\eta}$ VIA MATCHING MOMENTS:

$\bar{x} = 53 \Rightarrow 2809$

$s = 3.594 \Rightarrow s^2 = 1292$

$\frac{s^2}{\bar{x}^2} = \frac{1292}{2809} = 0.45995$

FROM MAGICAL b GRAPH

$b = 0.66$

$b = 1/\hat{\beta} \Rightarrow \hat{\beta} = 1.52$

$$\eta = \frac{\bar{x}}{\Gamma[1+b]} = \frac{\bar{x}}{\Gamma[1.66]} = \frac{53}{0.902} = 58.8$$

13. a. FOR WEIBULL

$$R(x) = e^{-(\frac{x}{\eta})^{\beta}}$$

THUS

$$\hat{R}(x) = e^{-(\frac{x}{\hat{\eta}})^{\hat{\beta}}}$$

AND

$$\hat{R}(1) = e^{-(\frac{1}{58.8})^{1.52}} = 0.99796$$

Pg IV - 242

15. $\theta_0 = 1200$ HRS

$n = 4$

a. ASSUME EXPONENTIAL

NON-REPLACEMENT FAILURE TERMINATED TEST

$r = 2$

$H_0 : \theta = \theta_0 = 1200$ HRS.

$H_1 : \theta < \theta_0$

$\alpha = P[\text{ACCEPTING } H_0 / H_1 \text{ IS TRUE}] = 0.05$

WE WILL ACCEPT H_0 IF

$$\hat{\theta} \geq \frac{\theta_0 \, \chi^2_{1-\alpha;2r}}{2r}$$

$$\chi^2_{1-\alpha;2r} = \chi^2_{0.95;4} = 0.711$$

$$\Rightarrow \hat{\theta} \geq \frac{1200 \times 0.711}{4} = 213.3$$

b. $H_0 : \theta = \theta_0 = 1200$

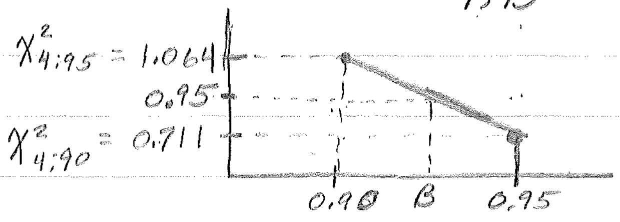
$H_1 : \theta = \theta_1 = 900$

FIND $\beta = P[\text{ACCEPTING } H_0 / \theta = \theta_1 = 900 \text{ HRS}]$

$$= P[\chi^2_{2r} \geq \frac{\theta_0}{\theta} \chi^2_{1-\alpha;2r}]$$

$$= P[\chi^2_4 \geq (\tfrac{12}{9})(0.711) = 0.95 \quad]$$

FROM TABLE D; $\chi^2_{4;95} = 0.711$ & $\chi^2_{4;90} = 1.064$



$$\frac{\beta - 0.9}{1.064 - 0.950} = \frac{0.95 - 0.9}{1.064 - 0.711} = \frac{0.05}{0.353}$$

$$\Rightarrow \beta = \frac{0.05}{0.353} \times 0.114 + 0.9$$

$$= 0.916$$

Pg IV-242

15. $\theta_0 = 1200$ HRS

$n = 4$

a. ASSUME EXPONENTIAL

NON-REPLACEMENT FAILURE TERMINATED TEST

$r = 2$

$H_0: \theta = \theta_0 = 1200$ HRS.

$H_1: \theta < \theta_0$

$\alpha = P[\text{ACCEPTING } H_0 / H_1 \text{ IS TRUE}] = 0.05$

WE WILL ACCEPT H_0 IF

$$\hat{\theta} \geq \frac{\theta_0 \chi^2_{1-\alpha;2r}}{2r}$$

$$\chi^2_{1-\alpha;2r} = \chi^2_{0.95;4} = 0.711$$

$$\Rightarrow \hat{\theta} \geq \frac{1200 \times 0.711}{4} = 213.3$$

b. $H_0: \theta = \theta_0 = 1200$

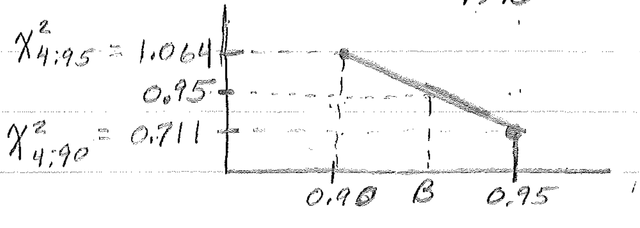
$H_1: \theta = \theta_1 = 900$

FIND $\beta = P[\text{ACCEPTING } H_0 / \theta = \theta_1 = 900 \text{ HRS}]$

$$= P\left[\chi^2_{2r} \geq \frac{\theta_0}{\theta} \chi^2_{1-\alpha;2r}\right]$$

$$= P\left[\chi^2_4 \geq \left(\frac{12}{9}\right)(0.711) = 0.95 \right]$$

FROM TABLE D; $\chi^2_{4;95} = 0.711$ & $\chi^2_{4;90} = 1.064$



$$\frac{\beta - 0.9}{1.064 - 0.950} = \frac{0.95 - 0.9}{1.064 - 0.711} = \frac{0.05}{0.353}$$

$$\Rightarrow \beta = \frac{0.05}{0.353} \times 0.114 + 0.9$$

$$= 0.916$$

(CONT.) Pg 242-3

c. $X_t = 600 + 900 + 2(900)$

$= 1500 + 1800 = 3300$

$\hat{\theta} = \frac{X_t}{r} = 1650$

FROM CRITERIA IN PART a : $\hat{\theta} \geq 213.3$

ACCEPT THE LOT

d. REQUIRED

$H_0 : \theta = \theta_0 = 1200$ HRS

$H_1 : \theta = \theta_1 = 300$ HRS

$\alpha = P[$REJECTING H_0 / H_0 IS TRUE$] = 0.05$

$\beta = P[$ACCEPTING H_0 / H_1 IS TRUE$] = 0.10$

TO MEET BOTH REQUIREMENTS, r MUST SATISFY

$$\frac{\chi^2_{\beta;2r}}{\chi^2_{1-\alpha;2r}} = \frac{\theta_0}{\theta_1}$$

OR $$\frac{\chi^2_{0.1,2r}}{\chi^2_{0.95,2r}} = 4$$

FROM TABLE D

$$\frac{\chi^2_{0.1;10}}{\chi^2_{0.95;10}} = 4.06$$

$\Rightarrow$ LET $2r = 10 \Rightarrow r = 5$

Pg IV-243

16. TIME TERMINATED TEST : $T = 300$ HRS

EXPONENTIAL ASSUMPTION

$n = 6$

a. NON-REPLACEMENT

$\alpha = 0.10$

$H_0: \theta = \theta_0 = 1800$ HRS

$H_1: \theta < \theta_0$

FOR NON-REPLACEMENT TEST:

$$1-\alpha = \sum_{k=0}^{r_0-1} \binom{n}{k} \left[1 - e^{-T/\theta_0}\right]^k \left[e^{-T/\theta_0}\right]^{(n-k)}$$

$$K=0: \binom{6}{0} \left[1 - e^{-\frac{300}{1800}}\right]^0 \left[e^{-\frac{300}{1800}}\right]^6$$

$$= e^{-\frac{6(300)}{1800}} = e^{-1} = 0.3679$$

$$K=1: \binom{6}{1} \left[1 - e^{-3/18}\right] \left[e^{-3/18}\right]^5$$

$$= 6\left[1 - e^{-3/18}\right]\left[e^{-15/18}\right] = \underline{0.4003}$$

$$0.7682$$

$$K=2: \binom{6}{2} \left[1 - e^{-3/18}\right]^2 \left[e^{-3/18}\right]^4$$

$$\frac{6 \cdot 5}{2}\left[1 - e^{-3/18}\right]^2 \left[e^{-2/3}\right] = \underline{0.1815}$$

$$0.9497$$

$\therefore$ LET $r_0 - 1 = 2 \Rightarrow r_0 = 3$

$1 - \alpha_{TRU} = 0.95 \Rightarrow \alpha_{TRU} \cong 0.05$

b. $H_0: \theta = \Theta_0 = 1800$

$H_1: \theta = \Theta_1 = 1200$

FIND $\beta = P[\text{ACCEPTING } H_0 / H_1 \text{ IS TRUE}]$

GIVEN $\alpha = 0.05$ AND $r_0 = 3$, THEN

$$\beta = \sum_{k=0}^{r_0-1} \binom{n}{k} \left[1 - e^{-T/\theta_1}\right]^k \left[e^{-T/\theta_1}\right]^{n-k}$$

$$k=0: \binom{6}{0}\left[1-e^{-T/\theta_1}\right]^0 \left[e^{-\frac{300}{1200}}\right]^6$$

$$= e^{-(0.25)6} = 0.2231$$

$$k=1: \binom{6}{1}\left[1 - e^{-0.25}\right] e^{-(0.25)5} = 0.3802$$

$$k=2: \binom{6}{2}\left[1 - e^{-0.25}\right]^2 e^{(-0.25)4}$$

$$15\left[1-e^{-0.25}\right]^2 e^{-1} = \underline{0.2700}$$

$$\Rightarrow \beta = 0.8733$$

c. X_i : 50, 90, 226, 298

$r = 4$ FAILURES

WE REJECT IF $r > r_0 = 3$

$\therefore$ REJECT

Pg IV-241-2

14. $n = 4$

NORMALLY DISTRIBUTED

a. $H_0: \mu = \mu_0 = 30{,}000$ $\alpha = 0.1$

$H_1: \mu \leq \mu_0 = 30{,}000$

TEST STATISTIC

$$t_{n-1} = \frac{\bar{X} - \mu_0}{S\sqrt{n}} = \frac{\bar{X} - 30{,}000}{S/2} = t_3$$

ACCEPT H_0 IF

$$t_3 > t_{1-\alpha; n-1} = t_{0.90;3}$$

$$t_{90;3} = {}^{-}t_{10;3} = {}^{-}1.638$$

$$\Rightarrow t_3 > {}^{-}1.638$$

b. $H_0: \sigma = \sigma_0 = 1000$ $\alpha = 0.05$

$H_1: \sigma > \sigma_0$

TEST STATISTIC:

$$\chi^2_{n-1} = \frac{(n-1)S^2}{\sigma_0^2} = \frac{3}{10^6} S^2 = \chi^2_3$$

ACCEPT H_0 IF

$$\chi^2_3 \leq \chi^2_{\alpha; n-1} = \chi^2_{0.05,3} = 7.815$$

c. X_i: 30,000; 35,000; 28,000; 32000

$\bar{X} = 31250$

$S = 298.6.$ $S^2 = 8.917 \times 10^6$

i. $t_3 = \frac{31250 - 30{,}000}{2986./2} = 0.837$

$t_3 = 0.837 > -1.638 \Rightarrow$ ACCEPT $H_0: \mu = \mu_0$

ii. $\chi^2_{n-1} = (3 \times 10^{-6})(8.917 \times 10^6) = 26.75$

$26.75 \nleq 7.815$

$\Rightarrow$ REJECT $H_0: \sigma = \sigma_0$

Pg. IV-244

17. FTT W/R , $r=6$, $n=10$ [EXPONENTIAL]

$H_0: \theta = \theta_0 = 200$; $\alpha = 0.05$

$H_1: \theta < \theta_0$

FROM TABLE 2B.1 IN H 108

Code B-6 $\frac{c}{\theta_0} = 0.436$

ACCEPT LOT IF $\hat{\theta} \geq \theta_0 \left(\frac{c}{\theta_0}\right) = 87.2$

$X_t = n x_r = (10)(194) = 1940$

$\hat{\theta} = \frac{X_t}{r} = \frac{1940}{6} = 323.33 > 87.2 \Rightarrow$ ACCEPT LOT

$\frac{\theta}{\theta_0} = \frac{180}{200} = \frac{9}{10} = 0.9$

EMPLOYING OC CURVE B6:

$P[\text{ACCEPTING} / \frac{\theta}{\theta_0} = 0.9] = 0.92$

Pg. IV-244

18. W/O REPLACEMENT

FTT: $r = 4$, $n = 20$

$H_0: \theta = \theta_0 = 900$; $\alpha = 0.25$

$H_1: \theta < \theta_0$

FROM H-108, TABLE 2B-1, Pg 2.28

CODE D-4 $\frac{c}{\theta_0} = 0.634$

ACCEPT LOT IF $\hat{\theta} \geq \theta_0 \left(\frac{c}{\theta_0}\right) = 900(0.634) = 570.6$

X_i : 10 , 330 , 435 , 655

$X_t = \sum X_i + (n-r) X_r$; $\hat{\theta} = \frac{X_t}{r} = 2977.5$

$\Rightarrow$ ACCEPT THE LOT

$H_0: \theta = \theta_0 = 900$

$H_1: \theta = \theta_1 = 360$

$$\frac{\theta_1}{\theta_0} = \frac{360}{900} = \frac{40}{100} = \frac{20}{50} = \frac{2}{5} = 0.4$$

FROM OC CURVE D4, $P_a = 0.12$

Pg IV - 247

24. $H_0 : \Theta = 300 = \Theta_0$ $\alpha = 0.01$

$H_1 : \Theta < 300$

FTT : r = 3 W REPLACEMENT

n = 15

FROM TABLE 2C-2

Code A-3 $\frac{T}{\Theta_0} = 0.029 \Rightarrow T = 8.70$

THE TEST WILL THUS BE TERMINATED UPON OCCURANCE OF 3^{RD} FAILURE OR AT 8.7 HRS, WHICHEVER COMES FIRST.

$X_F = 21, 38, 189$

ALL DATA COMES AFTER T = 8.7 HRS.

ACCEPT THE LOT

$\frac{\Theta}{\Theta_0} = \frac{100}{300} = 0.333$

FROM O.C. CURVE A_3 ; $P_A \approx 84.5\%$

$P_R = 1 - P_A = 1 - 0.845 = 0.155$

Pg IV - 248

26. T = 500 HRS   WITH REPLACEMENT

$H_0 : \theta = 5000 = \theta_0 \quad \alpha = 0.05$

$H_1 : \theta = 1000 = \theta_1 \quad \beta = 0.25$

$\frac{T}{\theta_0} = \frac{500}{5000} = \frac{1}{10}$; $\frac{\theta_1}{\theta_0} = \frac{1000}{5000} = \frac{1}{5}$

FROM TABLE 2C4 IN H108

$n = 8$, $r = 3$

∴ WE REJECT LOT IF 3 OF 8 TESTED ITEM FAILS PRIOR TO 500 HRS

TEST DATA:

X_f : 10, 42, 95, 286, 519

3^{RD} FAILURE OCCURED @ 95 HRS

$\Longrightarrow$ REJECT THE LOT

Pg IV-250

29. NON-REPLACEMENT SEQUENTIAL

$H_0: \theta = \theta_0 = 500$ HRS $\alpha = 0.05$

$H_1: \theta = \theta_1 = 100$ HRS $\beta = 0.10$

$\frac{\theta_1}{\theta_0} = \frac{1}{5} = 0.20$

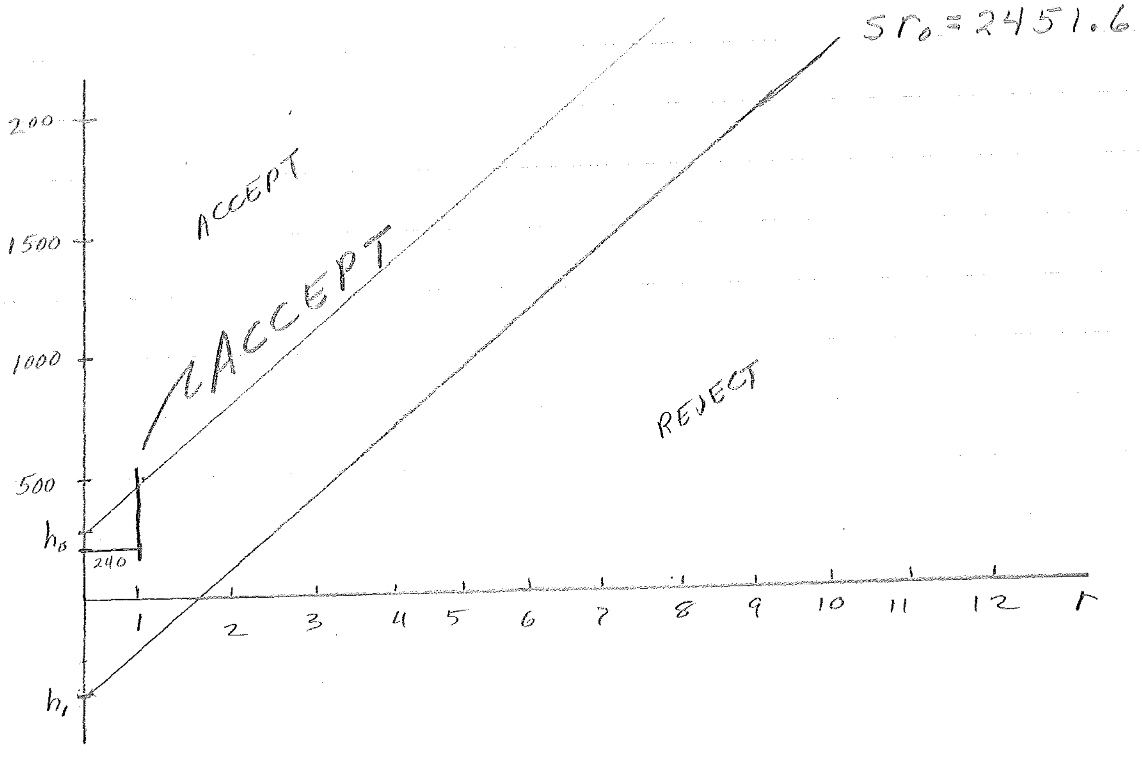
FROM TABLE 2A1, IN H-108, USE Code B-4

FROM TABLE 2D-1(b)

$r_0 = 12$ $\frac{h_0}{\theta_0} = 0.5805$ $\frac{h_1}{\theta_0} = 0.7453$ $\frac{s}{\theta_0} = 0.4086$

$h_0 = 290.25$ $h_1 = 372.65$ $s = 204.30$

$sr_0 = 2451.6$



$n = r_0 = 12$

$t_1 = (12)\,20 = 240$

$t_2 = (11)(50) + 20 = 570$

Pg. IV-250-1

31. $\beta = 2$

$t = 300$ HRS

$H_0: \mu = \mu_0 = 15{,}000 \quad \alpha = 0.05$

$H_1: \mu = \mu_0 = 6000 \quad \beta = 0.10$

$$\frac{100t}{\mu_1} = \frac{(100)(300)}{6000} = 5$$

$$\frac{100t}{\mu_0} = \frac{(100)(300)}{15{,}000} = 2$$

$n = 2{,}661$, $c = 2 \Leftarrow$ FROM TABLE 3B IN TR3

TEST DATA: $X_F = 101{,}290 \Rightarrow r = 2$

$c \geq r \Rightarrow$ ACCEPT LOT

Pg IV 252

34. $\beta = 2$

$t = 800$ $\alpha = 0.05$

$H_0: Z = 2\times10^{-5} = Z_0$

$H_1: Z = 5\times10^{-5} = Z_1$

$100\ Z_0 t = (100)(2\times10^{-5})(800) = 1.6$

$100\ Z_1 t = (100)(5\times10^{-5})(800) = 4.0$

FROM TABLE 3g IN TR4

$n = 838,\ C = 11$

DATA: 101, 301, 759 $\Rightarrow r = 3$

$C \geq r \Rightarrow$ ACCEPT LOT

[illegible]
comp [illegible]
is in your [illegible]
be accepted [illegible]
a sequential [illegible]
invitation [illegible]
true mean of 2000 hours [illegible]
are the failure times of [illegible]
all 100 components from [illegible]
suppliers should be accepted [illegible]

Supplier A

9
4
22
23
27
32
37
45
60
69

USE NON-REPLACEMENT
DON'T WORRY ABOUT TRUNCATION PTS.

$H_0: \theta = \theta_0 = 2000$ HRS $\quad \alpha = 0.1$

$H_1: \theta = \theta_1 = 1000$ HRS $\quad \beta = 0.1$

$n = 100$

$\frac{\theta_1}{\theta_0} = \frac{1}{2}$ → FROM TABLE 2A-1, IN H108, USE CODE C-11

TABLE 2D-1 IN H108 GIVES

$r_0 = 45 \quad \frac{h_0}{\theta_0} = 2.3503 \quad \frac{h_1}{\theta_0} = 2.3053 \quad \frac{s}{\theta_0} = 0.7024$

$h_0 = 4710.6 \quad h_1 = 4610.6 \quad s = 1404.8$

$s r_0 = 63{,}216$

A

	X_t
0	0
4	(99)(100) = 9900
22	(98)(22)+4 = 2160
23	(97)(23)+26 = 2257
27	(96)(27)+49 = 2641
32	(97)(32)+76 = 3180

REJECT

B

	X_t
1	(100)1 = 100
15	(99)(15)+1 = 1485
24	(98)(24)+16 = 2368
29	(97)(29)+40 = 2853
36	(96)(36)+69 = 3525
43	(95)(43)+105 = 4190
44	(94)(44)+148 = 4284

REJECT

C

	X_t
3	(3)(100) = 300
4	(4)(99)+3 = 399
42	(42)(98)+7 = 4123
113	(113)(97)+49 = 11,010

ACCEPT

Pg V-29

1. RIVIT SETTER STENGTH: $\mu_{S'} = 300^{\#}$ $\sigma_{S'} = 4$ LBS

RIVIT STRAIN: $\mu_S = 275$ $\sigma_S = 3$

RELIABILITY MARGIN: $\frac{\mu_D}{\sigma_D} = \frac{\mu_{S'} - \mu_S}{[\sigma_{S'}^2 + \sigma_S^2]^{\frac{1}{2}}} = \frac{25}{5} = 5$

$P_F = P[z > 5] = 0{,}000000\ 289$

$R = 0.999999711$

,0000 289 % OF RIVITS WILL FAIL

Pg V-29

2.a. STRAIN : $\mu_S = 20$ LBS $\sigma_S = 0.5$ LBS

STRENGTH: μ_S $\sigma_{S'} = 0.75$

RELIABILITY MARGIN = $\frac{\mu_{S'} - \mu_S}{[\sigma_{S'}^2 + \sigma_S^2]^{1/2}}$

$\sigma_D = [\sigma_{S'}^2 + \sigma_S^2]^{\frac{1}{2}} = 0.901388$

$\Rightarrow$ RELIABILITY MARGIN = $\frac{\mu_{S'} - 20}{0.901388}$

FIND $\mu_{S'}$ SUCH THAT

$P_F = P[Z > \frac{\mu_{S'} - 20}{0.901388}] = 0.005$

FROM CUM. NORMAL TABLE $Z_{0.005} = 2.58$

$\Rightarrow \frac{\mu_{S'} - 20}{0.901388} = 2.58$

OR $\mu_{S'} = (2.58)(0.901388) + 20 = 22.33$ LBS

b. FOR $\sigma_{S'} = 0.25$ LBS

$\sigma_D = 0.5590$

$P_F = P[Z > \frac{22.33 - 20}{0.5590} = \frac{2.33}{0.559} = 4.2] = 0.0000207$

$\Rightarrow R_{NEW} = 0.9999793$

Pg II-73

15. $\omega = 4$, $\rho = 1$

FIND $R(50)$, $H(50)$

$$R(50) = P[X < \ln 50]$$

$$= P\left[Z < \frac{\ln 50 - 4}{1} = 0.088\right]$$

$$= 1 - P[Z > 0.09]$$

$$= 1 - 0.464$$

$$= 0.536$$

NOW $h(x) = \dfrac{f(x)}{R(x)}$

$$h(50) = \frac{f(50)}{R(50)};$$

$$f(x) = \frac{f(z)}{\rho x R(x)} \qquad f(z) = f(.08) = 0.397$$

$$h(50) = \frac{0.397}{(1)(50)(0.536)} = 0.0148$$

4. An AMETA student wishes to determine the effectiveness of his automobile in meeting the objective of getting to class. The distance to class is 50 miles. Based on his experience, failures other than tire failures are so unlikely that he is willing to neglect these. His experience shows that each tire fails at a constant rate of $\lambda = 1/500$ failures per mile when in use. His policy is to always have four good tires and one spare when he leaves home. He allows sufficient time so that a tire change can be made during the trip. He will not repair flats and will not drive on a flat. Define the significant states for this automobile. Set up Availability, Dependability, and Capability matrices which will, when multiplied, yield the probability of getting to class. (Put in all numerical entries even though some are not necessary for computation under the stated conditions.) What is the probability of accomplishing the stated objective?

CLASS SOLUTION

STATES: 1. 0 FAILURES

2. 1 FAILURE

3. MORE THAN 1 FAILURE

$X = 50$ MILES $\quad \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \frac{1}{4}\lambda_5 \Rightarrow \lambda_5 = 4\lambda_1 = \frac{4}{500}$

OBJECTIVE: 1. DRIVE 50 MILES

AVAILABILITY MATRIX: $A = [a_{11}, a_{12}, a_{13}]$
$= [1 \quad 0 \quad 0]$

CAPABILITY $C = \begin{bmatrix} C_{11} \\ C_{21} \\ C_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

DEPENDABILITY: $D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$

$d_{11} = P[K = 0] = \frac{(\lambda x)^K e^{-\lambda x}}{K!} = e^{-0.4} = 0.670$

$d_{12} = P[K = 1] = \frac{(\lambda x)^K e^{-\lambda x}}{K!} = \frac{(0.4)^1 e^{-0.4}}{1!} = 0.268$

$d_{13} = 1 - d_{11} - d_{12} = 0.062$

$d_{21} = 0$

$d_{22} = P[K = 0] = 0.670$

$d_{23} = 1 - d_{21} - d_{22} = 0.330$

$d_{31} = 0$

$d_{32} = 0$

$d_{33} = 1$

$$\Rightarrow D = \begin{bmatrix} 0.670 & 0.268 & 0.062 \\ 0 & 0.670 & 0.330 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow [E] = [1 0 0] \begin{bmatrix} 0.670 & 0.268 & 0.062 \\ 0 & 0.670 & 0.330 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0.938$$

MY SOLUTION

OBJECTIVE: GETTING TO CLASS

STATES : 1 - CAR IS OPERATING WITH ONE SPARE

2 - CAR IS OPERATING WITH NO SPARE

3 - CAR HAS 2 FLATS

ASSUME : P[FAILURE AT START] = 0

P[CHANGING A FLAT TIRE] = 1

SYSTEM : 1 CAR

TIRES: A, B, C, D, S $\qquad \lambda_{SYS} = 4\lambda_{TIRE} = \frac{4}{500}$

AVAILABILITY: $A = [a_{11}, a_{12}, a_{13}]$ $\qquad \lambda_s X = 0.4$

$= [1 \quad 0 \quad 0]$

DEPENDABILITY: $R(50) = e^{-50\lambda} = e^{-\frac{50}{500}} = e^{-0.1} = 0.90$

USE POISSON: $\lambda X = 0.4$; $P[\underline{K} = k] = \frac{(\lambda x)^k e^{-\lambda x}}{k!}$

$d_{11} = P[0 \text{ FAILURE}] = \frac{(0.4)^0 e^{-0.4}}{0!} = e^{-0.4} = 0.670$

$d_{12} = P[1 \text{ FAILURE}] = \frac{(0.4)^1 e^{-0.4}}{1!} = 0.268$

$d_{13} = P[2 \text{ FAILURES}] = \frac{(0.4)^2 e^{-0.4}}{2!} = 0.054$

$d_{21} = 0$

$d_{22} = 0\,(?)$

$d_{23} = 0\,(?)$

$d_{31} = 0$

$d_{32} = 0$

$d_{33} = 0\,(?)$

$$D = \begin{bmatrix} 0.670 & 0.268 & 0.054 \\ 0 & 0(?) & 0 \\ 0 & 0(?) & 0(?) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow [E] = [1 \ 0 \ 0] \begin{bmatrix} .670 & .268 & .054 \\ 0 & 0(?) & 0 \\ 0 & 0(?) & 0(?) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= [0.670 \quad 0.268 \quad 0.054] \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= [0.938] \ \{= P[\underline{K} \leq 1]\}$$

OUTLINE SUMMARY

I. PROBABILITY DEFINITIONS

A. CLASSICAL: (APRIORI)

$$P[A] = \frac{a}{a+b}$$

a = POSSIBLE NUMBER OF SUCCESSES IN EVENT "A"

b = " " " FAILURES " " "

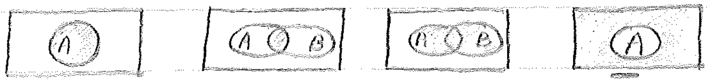
B. RELATIVE FREQUENCY: (POSTERIORI)

$$P[A] = \frac{m}{n}$$

WHERE m IS THE NUMBER OF SUCCESSES IN n TRIALS

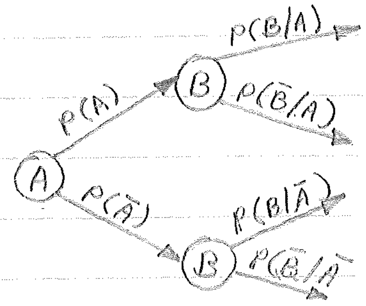
II. REPRESENTATIONS OF PROBABILITY EVENTS

A. VENN DIAGRAMS



A $A \cap B$ $A \cup B$ $\bar{A}$

B. TREE DIAGRAMS



III. BASIC PROBABILITY LAWS

A. $P[A] + P[\bar{A}] = 1$

B. $P[A \cap B] = P[A]\,P[B/A] = P[B]\,P[A/B]$

FOR INDEPENDENT EVENTS: $P[A \cap B] = P[A]\,P[B]$

C. $P[A \cup B] = P[A] + P[B] - P[A \cap B]$

FOR MUTUALLY EXCLUSIVE EVENTS:

$P[A \cup B] = P[A] + P[B]$

IV. PROBABILITY FUNCTIONS

(PROBABILISTIC STATEMENTS ABOUT UNKNOWN
x IN RESPECT TO RANDOM VARIABLE $\underline{X}$.)

A. DENSITY FUNCTION:

$$f(x) = P[x \leq \underline{X} \leq x + dx]$$

B. CUMMULATIVE DISTRIBUTION:

$$F(x) = P[x \leq \underline{X}] = \int_{-\infty}^{x} f(x)\,dx$$

C. RELIABILITY

$$R(x) = P[x > \underline{X}] = 1 - F(x) = \int_{x}^{\infty} f(x)\,dx$$

D. HAZARD FUNCTION:

$$h(x) = f(x)/R(x) = \frac{\text{FAILURES}}{\text{TIME}} \text{ AT TIME } x.$$

V. STATISTICAL MEASURES

FOR A SAMPLE n FROM POPULATION N

A. MEAN

1. TRUE MEAN: $\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$
2. SAMPLE MEAN: $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$

B. VARIANCE

1. TRUE VARIANCE:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2 = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - \mu^2\right]$$

2. SAMPLE VARIANCE

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{n}{n-1}\bar{X}^2\right]$$

3. EXPECTED VALUE

$$E[x] = \mu$$

$$E[(x-\mu)^2] = \sigma^2$$

VI. DISTRIBUTIONS

A. BINOMIAL

GIVEN n TRIALS WITH PROBABILITY OF SUCCESS p AND PROBABILITY OF FAILURE q = 1 - P, THE PROBABILITY OF X SUCCESSES IS

1. $f(x) = P[\underline{X} = x] = \binom{n}{x} p^x q^{n-x}$

   WHERE $\binom{n}{x} = \frac{n!}{x!(n-x)!}$
2. $F(x) = P[\underline{X} \leq x] = \sum_{m=0}^{x} \binom{n}{m} p^m q^{n-m}$
3. $R(x) = 1 - F(x) = \sum_{m=x+1}^{n} \binom{n}{m} p^m q^{n-m}$
4. $h(x) = f(x)/R(x)$
5. SAMPLE STATISTICS

   $\sigma^2 = npq \;:\; \mu = Np$

B. POISSON

GIVEN AN INFINITE POSSIBLE NUMBER OF OCCURANCES FROM A POPULATION WITH CONSTANT FAILURE RATE λ, THEN THE PROBABILITY OF k FAILURES IS

1. $f(x) = P[\underline{K} = k) = \frac{(\lambda x)^k e^{-\lambda x}}{k!}$

   WHERE X = INTERVAL OF INTEREST
2. $F(x) = P[\underline{K} \leq k] = \sum_{m=0}^{k} \frac{(\lambda x)^m e^{-\lambda x}}{m!}$
3. $R(x) = P[\underline{K} > k] = 1 - F(x)$
4. $h(x) = f(x)/R(x)$

NOTE: POISSON MAY BE USED TO APPROXIMATE THE BINOMIAL FOR SMALL λ AND BIG X.

C. EXPONENTIAL

CONSTANT HAZARD RATE: λ

1. $f(x) = \lambda e^{-\lambda x}$ $\quad \mu(x)$
2. $F(x) = 1 - e^{-\lambda x}$
3. $R(x) = e^{-\lambda x}$
4. $h(x) = \lambda$
5. MEAN: $\mu = \frac{1}{\lambda}$

VARIANCE: $\sigma^2 = \frac{1}{\lambda^2}$

D. WEIBULL: β = SHAPE; η = SCALING

1. $f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^{\beta}}$ $\quad \mu(x)$
2. $F(x) = 1 - e^{-(x/\eta)^{\beta}}$
3. $R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$
4. $h(x) = \left(\frac{\beta}{\eta}\right)\left(\frac{x}{\eta}\right)^{\beta-1}$
5. MEAN: $\mu = \eta\,\Gamma\left(\frac{1}{\beta} + 1\right)$

$$\sigma^2 = \eta^2\left[\Gamma\left(1 - \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

6. THE Γ FUNCTION:

$$\Gamma(x) = \int_0^{\infty} x^{n-1} e^{-x} dx$$

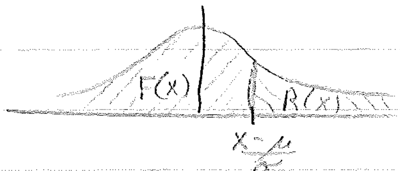
$$\Gamma(x+1) = x\Gamma(x)$$

E. NORMAL DISTRIBUTION

VARIANCE: σ^2; MEAN: μ

1. $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
2. $F(z)$ & $R(z)$ & $h(z)$ & $f(z)$ ARE TABLED: $z = \frac{x-\mu}{\sigma}$

$$F(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}z^2} dz = P\left[\frac{x-\mu}{\sigma} < z\right]$$



$$f(x) = \frac{1}{\sigma} f(z)$$

F. GAMMA DISTRIBUTION

α: SHAPE; λ: SCALE

1. $f(x) = \dfrac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$

IF α IS AN INTEGER, WE HAVE A POISSON

2. USE FOR CONSTANT HAZARD RATE WHERE α^{TH} EVENT CAUSES FAILURE

VII. CONDITIONAL RELIABILITY

IF A SYSTEM OPERATES SUCCESSFULLY TO TIME t_0, THE SYSTEM RELIABILITY FROM t_0 TO t_1 IS:

$$P[T > t_1 / T > t_0, t_1 > t_0] = \frac{R(t_1)}{R(t_0)}$$

VIII. SYSTEMS AND SUBSYSTEMS

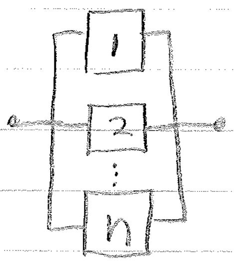
A. SERIES: ALL SYSTEMS NEEDED FOR SUCCESS:

FOR SUBSYSTEMS INDEPENDENDENT:

$$R_S = \prod_{i=1}^{n} R_i$$

FOR SUBSYSTEMS ALSO IDENTICAL: $R_S = R_i^{\,n}$

B. ACTIVE REDUNDANCY



1. SIMPLE: ONLY 1 ITEM NEEDED FOR SUCCESS

a. ALL ITEMS INDEPENDENT:

$$Q_S = 1 - R_S = \sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} (1 - R_i)$$

AND EQUAL: $Q_S = Q_i^{\,n}$

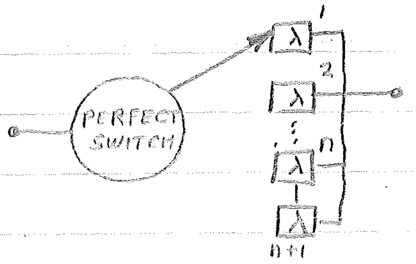
2. PARTIAL ACTIVE REDUNDANCY

k ITEMS NEEDED FOR SUCCESS

$$R_S = P[\underline{K} \geq k] = \sum_{x=k}^{n} \binom{n}{x} R^{x} Q^{n-x}$$

C. STANDBY REDUNDANCY

1. IDENTICAL INDEPENDENT EXPONENTIAL WITH PERFECT SWITCH

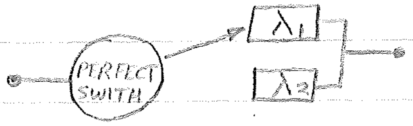


n = NUMBER OF FAILURES

$$R_S = P[K \le n]$$

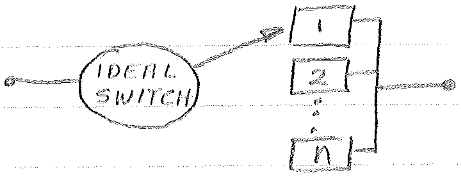
$$= \sum_{K=0}^{n} \frac{(\lambda x)^K e^{-\lambda x}}{K!}$$

2. TWO ELEMENT INDEPENDENT EXPONENTIAL WITH PERFECT SWITCH $(\lambda_1 \neq \lambda_2)$



$$R_S = \frac{\lambda_2 e^{-\lambda_1 x} - \lambda_1 e^{-\lambda_2 x}}{\lambda_2 - \lambda_1}$$

3. NON-TIME DEPENDENT MODEL WITH IDEAL SWITCH



ITEMS & SWITCH OPERATE INDEPENDENTLY

P_i = P[SUCCESSFUL OPERATION OF i^{TH} ELEMENT]

q_w = P[FAILURE TO SWITCH / IT IS APPROPRIATE]

q'_w = P[PREMATURE SWITCHING]

$q_i = 1 - P_i$; $p_w = 1 - q_w$; $p'_w = 1 - q'_w$

a. FOR $n = 2$

$$R_S = P_1 P'_w + P_1 q'_w P_2 + q_1 q_w P_2$$

b. FOR $n > 2$

USE TREE DIAGRAM
($n = 3$ GIVEN IN TEXT)

D. APPORTIONMENT

1. SIMPLE CASE

FOR n IDENTICAL INDEPENDENT ELEMENTS,

$$R_i = (R_s)^{1/n}$$

WHERE R_i & R_s ARE RESPECTIVELY ELEMENT AND SYSTEM RELIABILITIES

2. AGREE METHOD (K EXPONENTIAL INDEPENDENT SUBSYSTEMS

$i = 1, 2, 3, \ldots, k$

t = REQUIRED MISSION TIME FOR SYSTEM

t_i = REQUIRED MISSION TIME FOR i^{TH} SUBSYSTEM

w_i = IMPORTANCE FACTOR = P[i^{TH} SUBSYSTEM FAILURE WILL CAUSE SYSTEM FAILURE.

n_i = # OF MODULES IN i^{TH} SUBSYSTEM

$N = \sum_{i=1}^{k} n_i$ = TOTAL # OF MODULES

$R^*(t)$ = REQUIRED SYSTEM RELIABILITY

$R_i^*(t_i)$ = RELIABILITY APPORTIONED TO i^{TH} SUBSYSTEM

THEN

$$\theta_i = \frac{-t_i}{\ln\left[1 - \frac{1}{w_i}\left\{1 - (R^*(t))^{n_i/N}\right\}\right]}$$

$$\cong \frac{N w_i t_i}{n_i\left[-\ln R^*(t)\right]}$$

AND

$$R_i^*(t_i) = e^{-t_i/\theta_i}$$

$$R(t) = \prod_{i=1}^{k}\left[1 - w_i\left\{1 - R_i(t_i)\right\}\right]$$

3. ARINC METHOD (FOR EXPONENTIAL SERIES SYSTEM)

GIVEN: λ_i = FAILURE RATE FOR EACH SUBSYSTEM

λ^* = DESIRED SYSTEM FAILURE RATE $\leq \sum_{i=1}^{n} \lambda_i^*$

n = # OF SUBSYSTEMS

THEN $\lambda = \sum_{i=1}^{n} \lambda_i$ = SYSTEM FAILURE RATE

$w_i = \lambda_i / \lambda$ = WEIGHTING FACTOR

THE APPORTIONED FAILURE RATE FOR i^{TH} SUBSYSTEM, λ_i^*, IS

$$\lambda_i^* \leq w_i \lambda^*$$

$R_i^*(x) = e^{-\lambda_i^* x}$ = APPORTIONED RELIABILITY OF i^{TH} SUBSYSTEM

$R^*(x) = e^{-\lambda^* x}$ = " " " SYSTEM

4. MINIMAZATION OF EFFORT (SERIES SYSTEM)

GIVEN: R_i = PRESENT RELIABILITY OF i^{TH} SUBSYSTEM

R^* = DESIRED SYSTEM RELIABILITY = $\prod_{i=1}^{n} R_i^*$

n = # OF SUBSYSTEMS

THEN: 1. ORDER SUBSYSTEM RELIABILITIES: $R_1 \leq R_2 \leq \ldots \leq R_n \leq R_{n+1} = 1$

2. LET K_0 = MAXIMUM VALUE OF j SUCH THAT

$$R_j < \left[R^* / \sum_{i=j+1}^{n+1} R_i \right]^{1/j} = r_j$$

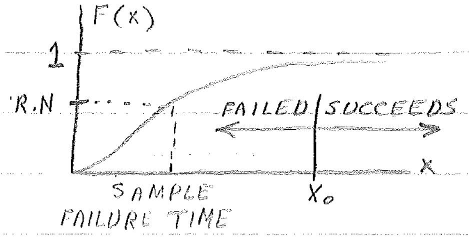
3. LET $R_0^* = \left[R^* / \sum_{i=j+1}^{n+1} R_j \right]^{1/K_0}$

4. LET SUBSYSTEMS 1 THRU K_0 HAVE APPORTIONED RELIABILITIES $R_j^* = R_0^*$.

LET K_0+1 TO n HAVE RELIABILITIES $R_j^* = R_j$

E. MONTE CARLO SIMULATION

GIVEN $F(x) = \int_{-\infty}^{x} f(x)\,dx$; $0 \leq F(x) \leq 1$



ESTIMATE OF $R(X_0) = \dfrac{\#\text{SUCCESSES}}{\#\text{SUCCESSES} + \#\text{FAILURES}}$

IX. RELIABILITY DEMONSTRATION AND TESTING

A. NON-PARAMETRIC

1. BINOMIAL

n SAMPLES PLACED ON TEST FOR TIME T

r FAIL, $d = n - r$ DO NOT FAIL

a. POINT ESTIMATE : $\hat{R}(t) = \frac{d}{n}$

d. CONFIDENCE INTERVALS: $(1-\alpha)$ CONFIDENT

→ $L \leq R(T) \leq U$ ⇒ TABLE K_1

→ $L \leq R(T)$ ⇒ TABLE K_2

→ $R(T) \leq U$ ⇒ USE r INSTEAD OF d

SUBTRACT TABLED VALUE (K_2) FROM 1

2. ORDER STATISTIC APPROACH

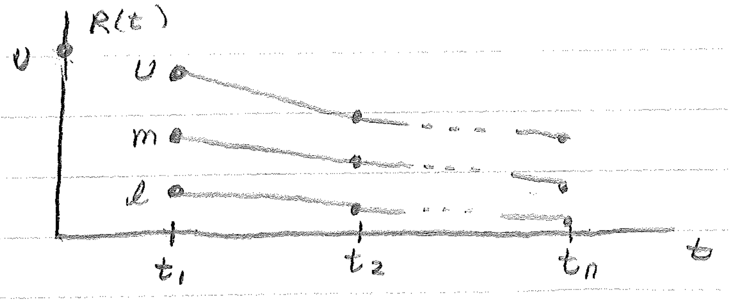
TEST n ITEMS UNTIL ALL HAVE FAILED

t_i = TIME OF i^{TH} FAILURE

TABLE THE FOLLOWING, WITH t_i ORDERED:

t_i	$\hat{F}_m(t_i)$	$\hat{F}_U(t_i)$	$\hat{F}_\ell(t_i)$	$\hat{R}_m$	$\hat{R}_U$	$\hat{R}_\ell$
	MEDIAN FROM TABLE H_1	UPPER FROM TABLE H_2	LOWER FROM TABLE H_3	$1-\hat{F}_m$	$1-\hat{R}_\ell$	$1-F_\ell$

GRAPH AS FOLLOWS



RELIABILITY ESTIMATES AND CONFIDENCE INTERVALS MAY BE TAKEN DIRECTLY FROM THE ABOVE LINEAR APPROXIMATION

B. PARAMETRIC TECHNIQUES

1. GRAPHICAL TECHNIQUES

IN ALL CASES: - ALL n ITEMS ARE TESTED TO FAILURE

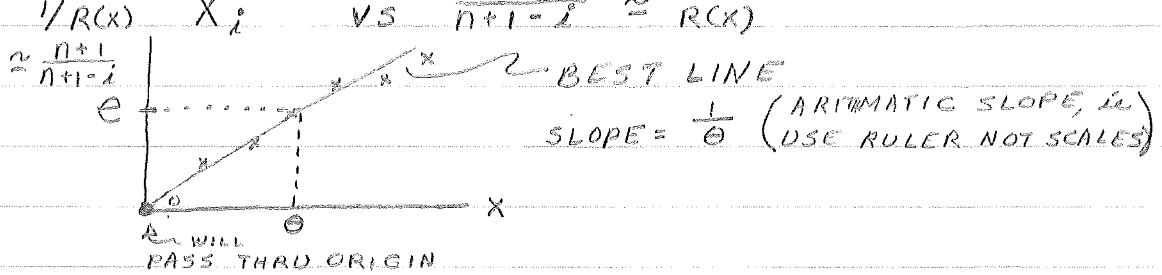
- ORDER FAILURE TIMES SUCH THAT

$$X_1 \leq X_2 \leq \ldots \leq X_i \leq \ldots \leq X_n$$

a. EXPONENTIAL

GRAPH ON EXPONENTIAL PAPER:

X_i VS $\frac{n+1}{n+1-i} \simeq \frac{1}{R(x)}$

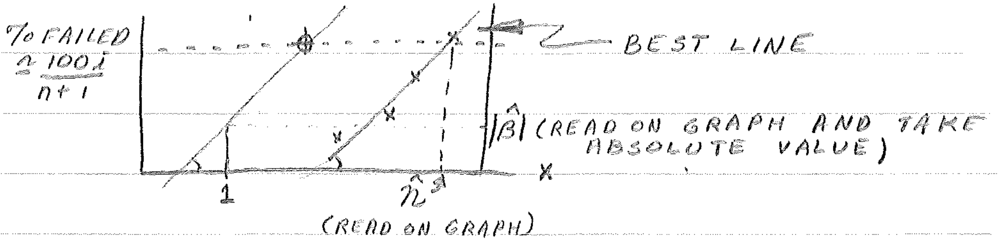


SLOPE = $\frac{1}{\theta}$ (ARITHMATIC SLOPE, i.e. USE RULER NOT SCALES)

b. WEIBULL

GRAPH ON WEIBULL PAPER

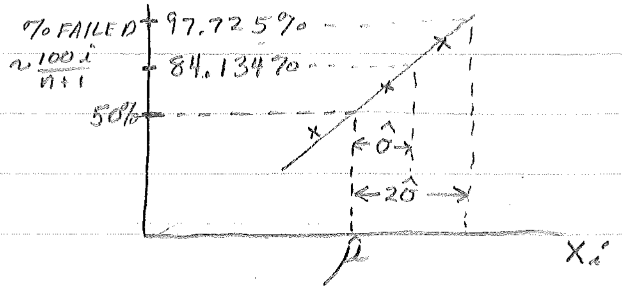
X_i VS $\frac{100i}{n+1}$ ~ % FAILED



c. NORMAL

GRAPH ON NORMAL PAPER

X_i VS $\frac{100i}{n+1}$ ~ % FAILED



C. GOODNESS OF FIT TEST (n ITEMS, ALL FAILED)

1. χ^2 TEST (NEED LARGE $n \sim (>30)$)
   - a. ASSUME A DISTRIBUTION
   - b. CHOOSE $\alpha = P[\text{REJECTION}/\text{TRUE DISTRIBUTION}]$
     (OPTIMALLY, α IS LARGE)
   - c. GROUP DATA INTO i INTERVALS
   - d. O_i = # SAMPLES IN i^{TH} INTERVAL
   - e. COMPUTE $E[\text{SAMPLES IN INTERVAL}] = E_i$
     (RULE OF THUMB: $2.5 < E_{i_{MINIMUM}} \le 5$)
     IT MAY BE NECESSARY TO COMBINE INTERVALS
   - f. COMPUTE: $\chi^2 = \sum_{i=1}^{n} \frac{1}{E_i}[O_i - E_i]^2$
   - g. REJECT DISTRIBUTION IF
     $$\chi^2 > \chi^2_{\alpha; n-p-1}$$
     WHERE P=# OF DISTRIBUTION PARAMETERS TESTED
2. KOLMOGOROV-SMIRNOV TEST (OK FOR SMALL n)
   - a. ASSUME UNDERLYING DISTRIBUTION AND COMPUTE PARAMETERS FROM FAILURE DATA.
   - b. CHOOSE $\alpha = P[\text{REJECTION}/\text{TRUE DISTRIBUTION}]$ (LARGE)
   - c. ORDER STATISTICS: $x_1 \le x_2 \le \ldots \le x_i \le \ldots \le x_n$
   - d. COMPUTE $\hat{F}(x_i) = \frac{i}{n}$
   - e. COMPUTE $F(x_i) = P[X < x_i]$
   - f. FIND MAXIMUM VALUE OF $d_i = d_{MAX}$ WHERE
     $$d_i = |F(x_i) - \hat{F}(x_i)|$$
   - g. REJECT DISTRIBUTION IF
     $$d > d_{\alpha; n}$$

O. PARAMETER ESTIMATION

1. EXPONENTIAL $[X_t = \text{TEST TIME}, \ \hat{\Theta} = \frac{X_t}{r}]$

a. FAILURE TERMINATED $[t_r = \text{TIME OF } r^{TH} \text{(LAST) FAILURE}]$

n ITEMS TESTED UNTIL r FAIL

i. POINT ESTIMATES

- WITHOUT REPLACEMENT

$$X_t = (n-r)X_r + \sum_{i=1}^{r} X_i \Rightarrow \hat{\Theta} = \frac{X_t}{r} = \frac{1}{r}\left[(n-r)X_r + \sum_{i=1}^{r} X_i\right]$$

- WITH REPLACEMENT

$$X_t = n X_r \Rightarrow \hat{\Theta} = \frac{X_t}{r} = \frac{n}{r} X_r$$

ii. CONFIDENCE INTERVAL $[(1-\alpha) \text{ CONFIDENCE}]$

$$-\Theta: \frac{2r\hat{\Theta}}{\chi^2_{\alpha/2;2r}} \leq \Theta \leq \frac{2r\hat{\Theta}}{\chi^2_{1-\frac{\alpha}{2};2r}}$$

$$-R(x): e^{-x\left[\frac{\chi^2_{\alpha/2;2r}}{2r\hat{\Theta}}\right]} \leq R(x) \leq e^{-x\left[\frac{\chi^2_{1-\frac{\alpha}{2};2r}}{2r\hat{\Theta}}\right]}$$

b. TIME TERMINATED $[T = \text{TERMINATION TIME}]$

i. POINT ESTIMATES

- WITHOUT REPLACEMENT

$$X_t = \sum_{i=1}^{r} X_i + (n-r)T \Rightarrow \hat{\Theta} = \frac{1}{r}\left[(n-r)T + \sum_{i=1}^{r} X_i\right]$$

- WITH REPLACEMENT

$$X_t = nT \Rightarrow \hat{\Theta} = \frac{n}{r} T$$

ii. CONFIDENCE INTERVALS $[(1-\alpha) \text{ CONFIDENCE}]$

$$-\Theta: \frac{2r\hat{\Theta}}{\chi^2_{\frac{\alpha}{2};2r+2}} \leq \Theta \leq \frac{2r\hat{\Theta}}{\chi^2_{1-\frac{\alpha}{2};2r}}$$

$$-R(x): e^{-x\left[\frac{\chi^2_{\frac{\alpha}{2};2r+2}}{2r\hat{\Theta}}\right]} \leq R(x) \leq e^{-x\left[\frac{\chi^2_{1-\frac{\alpha}{2};2r}}{2r\hat{\Theta}}\right]}$$

2. NORMAL

TEST n ITEMS UNTIL ALL FAIL

a. POINT ESTIMATES

→ $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$

→ $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2$

→ $\hat{R}(x) = P[\underline{X} > x] = P[Z > \frac{x-\bar{X}}{S}]$ ⇐ TABLE C

b. CONFIDENCE INTERVALS:

→ $\mu: \bar{X} \pm t_{\alpha/2;n-1} \times (\frac{S}{\sqrt{n}})$

→ $\sigma^2: \frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2};n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2};n-1}}$

→ $R(x): \hat{R}(x) \pm Z_{\alpha/2}\sqrt{V[\hat{R}(x)]}$: $Z_{\alpha/2} \rightarrow$ TABLE C

$V[\hat{R}(x)] = \frac{1}{n} f^2[Z = \frac{x-\bar{X}}{S}] \times [1 + \frac{1}{2}(\frac{x-\bar{X}}{S})^2]$

$f(Z) \rightarrow$ TABLE G

3. WEIBULL

a. POINT ESTIMATES

i. MATCHING MOMENTS

1. FROM STATISTICS, COMPUTE $\bar{X}$ AND S, THEN $\frac{S^2}{\bar{X}^2}$
2. FIND $b = \frac{1}{\beta}$ ON SOLUTION CURVE
3. $\hat{\eta} = \frac{\bar{X}}{\Gamma(1+b)}$
4. $\hat{R}(x) = e^{-(\frac{x}{\hat{\eta}})^{\hat{\beta}}}$

ii. MAXIMUM LIKLIHOOD METHOD

1. ASSUME $\hat{\beta}$
2. COMPUTE $\hat{\eta} = e^{-\frac{1}{\hat{\beta}}\ln\frac{1}{n}\sum_{i=1}^{n} x_i^{\hat{\beta}}}$
3. COMPUTE $\hat{\beta}_1 = \frac{n}{\sum_{i=1}^{n}(\frac{x_i}{\hat{\eta}})^{\hat{\beta}}\ln x_i - \sum_{i=1}^{n}\ln x_i}$
4. IF $\hat{\beta}_1 = \hat{\beta}$, WE IS DONE

IF $\hat{\beta}_1 \neq \beta$, LET $\hat{\beta}_{NEW} = \frac{1}{3}[\hat{\beta} + 2\hat{\beta}_1]$

REPEAT STEPS 2, 3, & 4

b. CONFIDENCE INTERVALS

$R(x): \hat{R}(x) \pm Z_{\alpha/2} \sqrt{V[R(x)]}$

$V[R(x)] \rightarrow$ NOT COMPUTABLE BY MORTALS

c. WITH β KNOWN

FAILURE REPLACEMENT TEST WITHOUT REPLACEMENT

$$\hat{R}(x) = e^{\left[\frac{-x^{\beta} r}{\sum_{i=1}^{r} X_i^{\beta} + (n-r) X_r^{\beta}}\right]}$$

$$e^{\frac{-x^{\beta}\chi^2_{\alpha/2;\,2r}}{2\left[\sum_{i=1}^{r} X_i^{\beta} + (n-r)X_r^{\beta}\right]}} \le R(x \le e^{\frac{-x^{\beta}\chi^2_{1-\alpha/2;\,2r}}{2\left[\sum_{i=1}^{r} X_i^{\beta} + (n-r)X_r^{\beta}\right]}}$$

E. TEST OF HYPOTHESIS

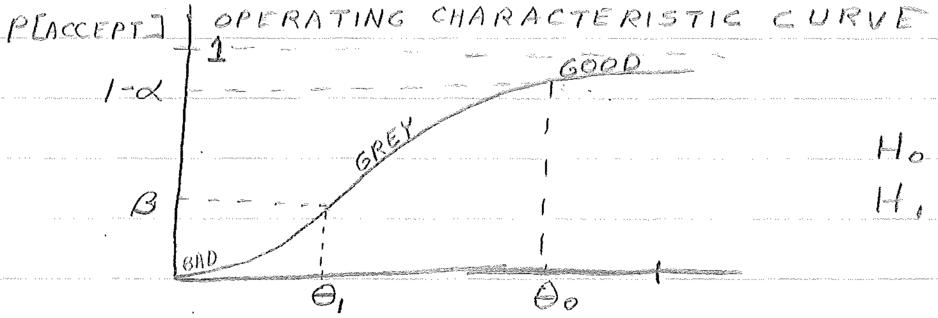
1. RISKS

$\alpha = P[$REJECTING H_0 / H_0 IS CORRECT$] \rightarrow$ PRODUCER'S RISK

$1-\alpha = P[$ACCEPTING H_0 / H_0 IS CORRECT$]$

$\beta = P[$ACCEPTING H_0 / H_0 IS INCORRECT$] \rightarrow$ CUSTOMER RISK

$1-\beta = P[$REJECTING H_0 / H_0 IS INCORRECT$]$ (H_1 IS CORRECT)



$H_0: \theta = \theta_0$

$H_1: \theta = \theta_1$

2. EXPONENTIAL

a. FAILURE TERMINATED TESTING

i. TEST FORMAT

$H_0 : \theta = \Theta_0$ α RISK

$H_1 : \Theta < \Theta_0$

- COMPUTE TEST STATISTIC $\hat{\Theta} = \frac{X_T}{r}$

- ACCEPT H_0 IF

ii. OPERATING CHARACTERISTIC CURVE FROM

$$P[\text{ACCEPTANCE}/\theta] = P[\chi^2_{2r} ; \tfrac{\Theta_0}{\Theta} \chi^2_{1-\alpha; 2r}]$$

iii. GIVEN α AND β, r MUST SOLVE

$$\frac{\chi^2_{\beta; 2r}}{\chi^2_{1-\alpha; 2r}} = \frac{\Theta_0}{\Theta_1} \quad \text{WHERE} \begin{cases} H_0 : \Theta = \Theta_0 \\ H_1 : \Theta = \Theta_1 \end{cases}$$

b. TIME TERMINATED TESTING (@ T)

i. TEST FORMAT

$H_0 : \Theta = \Theta_0$ α

$H_1 : \Theta < \Theta_0$

- ACCEPT H_0 IF $r < r_0$ WHERE r_0 SOLVES

FOR REPLACEMENT: $1 - \alpha = \sum_{K=0}^{r_0 - 1} \frac{1}{K!} \left[\left(\frac{nT}{\Theta_0} \right)^K e^{-\left(\frac{nt}{\Theta_0} \right)} \right]$

FOR W/O REPLACEMENT: $1 - \alpha = \sum_{K=0}^{r_0 - 1} \binom{n}{k} \left[1 - e^{-\left(\frac{T}{\Theta_0} \right)} \right]^k \left[e^{-\frac{T}{\Theta_0}} \right]^{n-k}$

ii. FOR O.C. CURVES:

- REPLACE: $P_a = P[r < r_0] = \sum_{K=0}^{r_0 - 1} \frac{1}{K!} \left(\frac{nT}{\Theta} \right)^K e^{-\left(\frac{nT}{\Theta} \right)}$

- W/O REPLACE: $P_a = P[r < r_0] = \sum_{K=0}^{r_0 - 1} \binom{n}{k} \left[1 - e^{-\left(\frac{T}{\Theta} \right)} \right]^k \left[e^{-\left(\frac{T}{\Theta} \right)} \right]^{n-k}$

iii. FOR $H_0 : \Theta = \Theta_0 (\alpha)$; $H_1 : \Theta = \Theta_1 (\beta)$ r_0 MUST SATISFY THE α CRITERIA IN (i) AND

- REPLACE: $\sum_{K=0}^{r_0 - 1} \frac{1}{K!} \left(\frac{nT}{\Theta_1} \right)^K e^{-\left(\frac{nT}{\Theta_1} \right)} = \beta$

- W/O REPLACE: $\sum_{K=0}^{r_0 - 1} \binom{n}{k} \left[1 - e^{-\left(\frac{T}{\Theta_1} \right)} \right]^k \left[e^{-\frac{T}{\Theta_1}} \right]^{n-k}$

3. NORMAL (n ITEMS, ALL FAIL)
   a. CONCERNING μ
      $H_0 : \mu = \mu_0$
      $H_1 : \mu < \mu_0$
      - TEST STATISTIC: $t_{n-1} = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}$
      - ACCEPT H_0 IF $t_{n-1} > t_{1-\alpha; n-1}$
   b. CONCERNING σ
      $H_0 : \sigma = \sigma_0$
      $H_1 : \sigma > \sigma_0$
      - TEST STATISTIC: $\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma_0^2}$
      - ACCEPT H_0 IF $\chi^2_{n-1} \leq \chi^2_{\alpha; n-1}$
   c. GIVEN α & β, n MUST SATISFY

$$\frac{\chi^2_{\alpha; n-1}}{\chi^2_{1-\beta; n-1}} = \frac{\sigma_1^2}{\sigma_0^2}$$

4. WEIBULL (NOT COVERED)

F. LIFE TESTING

1. EXPONENTIAL (H-108)
   - i. FTT
     - → FIND ACCEPTANCE REGION & O.C. CURVE

       GIVEN α, θ_0, r

       Pg. 2.28, TABLE 2B-1

       READ $\frac{C}{\theta_0}$ AND "Code"
         - ACCEPT LOT IF $\hat{\theta} \geq \theta_0 \left(\frac{C}{\theta_0}\right)$
         - O.C. CURVE GIVEN BY CODE
     - → FIND E[T] FOR FTT WITHOUT REPLACEMENT

       GIVEN n, r, AND θ_0

       Pp. 2.32-37, TABLE 2B-2a OR Pg. 2.38, TABLE 2B-2b
         - READ $\frac{T}{\theta_0}$
     - → FIND ACCEPTANCE REGION AND r

       GIVEN $\alpha, \beta, \theta_0, \theta_1 \xrightarrow{\text{COMPUTE}} \theta_1/\theta_0$

       Pg. 2.41, TABLE 2B-5

       READ r AND $\frac{C}{\theta_0}$
         - STOP TEST AFTER r FAILURES
         - ACCEPT LOT IF $\hat{\theta} \geq \theta_0 \left(\frac{C}{\theta_0}\right)$
   - ii. TTT
     - → FIND T AND O.C. CURVE

       GIVEN $\alpha, \theta_0, r, m \xrightarrow{\text{COMPUTE}} n = mr$
       - WITHOUT REPLACEMENT: Pg. 2.44-46, TABLE 2C-1
       - WITH REPLACEMENT: Pg. 2.47-49, TABLE 2C-2

       READ T/θ_0 AND "Code"
         - T = TEST TIME = $\theta_0 \left(\frac{T}{\theta_0}\right)$
         - O.C. CURVE FROM "CODE"

→ FIND r AND n

GIVEN $T, \theta_0, \theta_1, \alpha, \beta \xrightarrow{\text{COMPUTE}} \frac{T}{\theta_0}, \frac{\theta_1}{\theta_0}$

- WITHOUT REPLACEMENT: Pg. 2.51-2, TABLE 2C-3
- WITH REPLACEMENT: Pg. 2.53-4, TABLE 2C-4

READ n AND r [ACCEPT LOT IF $r_{OBS} \leq r$]

→ FIND n <u>WITHOUT</u> <u>REPLACEMENT</u>

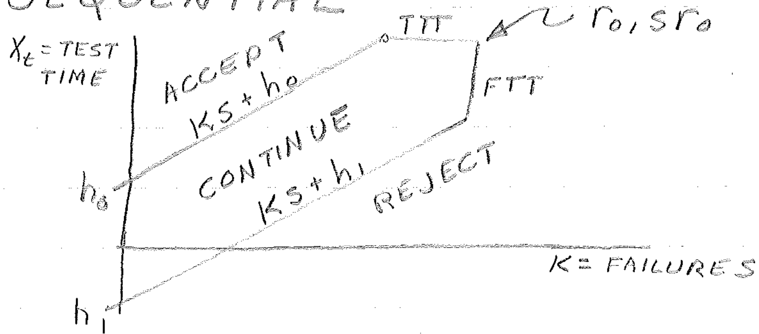
GIVEN $p_0, p_1, \alpha, \beta \xrightarrow{\text{COMPUTE}} \frac{p_1}{p_0} \ni p$ = PROPORTION FAILED

Pg 2.55, TABLE 2C-5

READ $\left(\frac{p}{p_0}\right)$

n IS CLOSEST INTEGER $\geq p_0\left(\frac{p}{p_0}\right)$

iii. SEQUENTIAL



GIVEN $\theta_0, \theta, \alpha, \beta$

- ANALYTICALLY

$$h_0 = \left[\frac{1}{1/\theta_1 - 1/\theta_0}\right] \ln\left[\frac{1-\alpha}{\beta}\right]$$

$$h_1 = \left[\frac{1}{1/\theta_1 - 1/\theta_0}\right] \ln\left[\frac{\alpha}{1-\beta}\right]$$

$$s = \left[\frac{1}{1/\theta_1 - 1/\theta_0}\right] \ln\left(\theta_0/\theta_1\right)$$

- FROM HANDBOOK

USE Pg 2-2, TABLE 2A1 TO FIND "Code"

USE CORRESPONDING PLAN ON Pp. 2.63-5, TABLE 2.63

READ r_0, h_0/θ_0, h_1/θ_0, s/θ_0

EXPECTED WAITING TIME FORMULAS FROM REMAINING ENTRIES GIVEN ON

Pg. 2.58 - W/o REP. AND Pg. 2.59 w/ REPL.

2. WEIBULL [$\alpha = 0.05$, $\beta = 0.10$, β_w IS KNOWN]

a. T-R-3 - MEAN LIFE CRITERION

- GIVEN μ_0, μ_1, t = TEST TERMINATION TIME, r
- FIND ACCEPTABLE REGION
  1. ENTER TABLE 3 (Pp. 28-36) WITH
     β_w (OR NEXT LARGEST) AND $\frac{100t}{\mu_1}$ (OR NEXT SMALLEST)
  2. READ n AND C   AND $\frac{100t}{\mu_0}$ (PAREN).
  3. ACCEPT LOT IF $r \leq C$

b. TR-4 - HAZARD RATE CRITERION

- GIVEN Z_0, Z_1, t, r
- FIND ACCEPTANCE REGION
  1. ENTER TABLE 3 (Pp. 46-56) WITH
     β_w, $100t\,Z_1$ (ROW), $100t\,Z_0$ (PAREN)
  2. READ n AND C
  3. ACCEPT LOT IF $r \leq C$

c. TR-6 - RELIABLE LIFE CRITERION

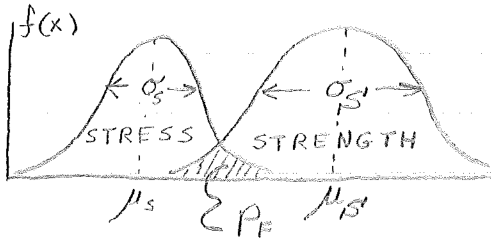
- GIVEN $\rho_0, \rho_1, r, t \ni \rho$ = RELIABLE LIFE
- FIND ACCEPTANCE REGION
  1. ENTER TABLE 3 (Pp. 37-59) WITH
     β_w, $\frac{100t}{\rho_1}$ (ROW), $\frac{100t}{\rho_0}$ (PAREN)
  2. READ n AND C
  3. ACCEPT LOT IF $r \leq C$

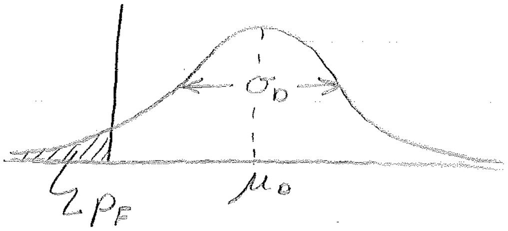
3. NORMAL - USE MIL-STD 414

4. NON PARAMETRIC - USE MIL-STD-105D

X. STRESS STRENGTH ANALYSIS

A. NORMAL STRESS-STRENGTH DISTRIBUTIONS



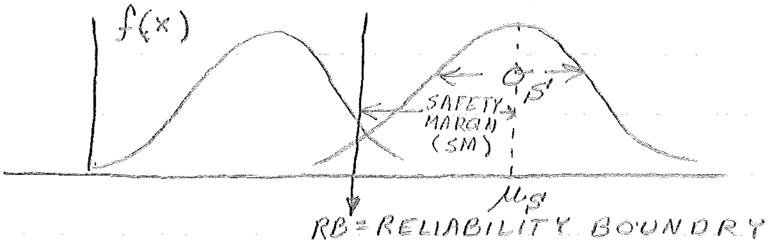


$\sigma_D^2 = \sigma_s^2 + \sigma_{S'}^2$ $\quad$ $\mu_D = \mu_{S'} - \mu_s$

RELIABILITY MARGIN = RM = $\frac{\mu_D}{\sigma_D}$

$P_F = P[Z > \frac{\mu_D}{\sigma_D} = Z_\alpha] = \alpha$: $R = 1 - P_F$

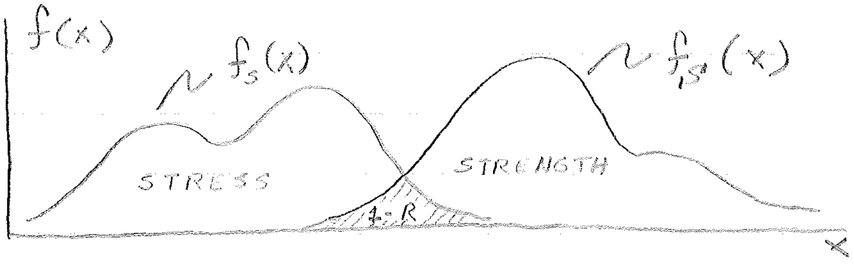
B. RELIABILITY BOUNDRY (SAFETY MARGIN)



SM = SAFETY MARGIN = $\frac{\mu_{S'} - RB}{\sigma_{S'}}$

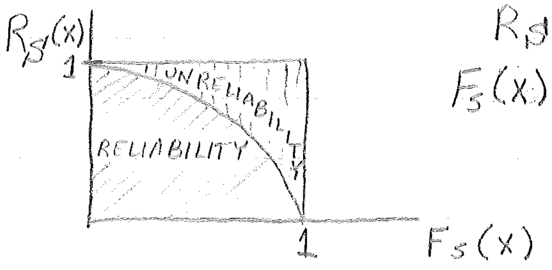
= SM IN STANDARD DEVIATIONS

C. GENERAL GRAPHICAL METHOD



IT CAN BE SHOWN THAT

$$R = \int_0^1 R_{S'}(x)\, d F_s(x)$$



$$R_{S'}(x) = 1 - \int_{-\infty}^{x} f_{S'}(x) dx$$

$$F_s(x) = \int_{-\infty}^{x} f_s(x) dx$$

XI. MAINTAINABILITY

A. DEFINITION

$M(x) = P[\mathbb{X} < x] = P[$SYSTEM CAN BE RESTORED BEFORE TIME $x]$

INDICES: MTTR → MEAN TIME TO RESTORE $= \frac{\text{TOTAL REPAIR TIME}}{\text{\# REPAIRS}}$

M_{MAX} → 90TH OR 95% PERCENTILE : $P[\mathbb{X} \leq M_{MAX}] = .9$

M_{50} → MEDIAN TIME OF RESTORATION

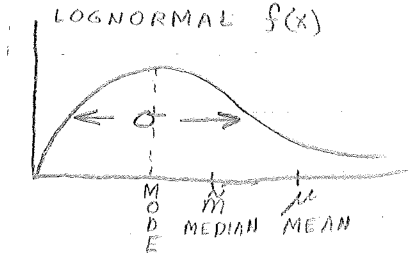
B. AVAILABILITY

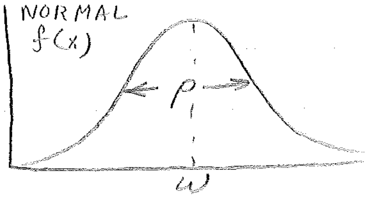
1. INHERENT: $A_i = \frac{\text{OPERATING TIME}}{\text{OPER. TIME + ACTIVE DOWNTIME}} = \frac{MTBM}{MTBM + M_t}$
2. OPERATIONAL: $A_o = \frac{\text{OPERATING TIME}}{\text{OPER. TIME + TOTAL DOWNTIME}} = \frac{MTBM}{MTBM + MDT}$

$A_o \leq A_i$; $M_t = MTTR_{CORR}$ $MDT = MTTR_{ACTV+DELAY}$

C. LOGNORMAL MODEL

$\ln \mathbb{X}$ IS NORMALLY DISTRIBUTED: $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln x - \mu}{\sigma}\right]^2}$ $\mu(x$





$$M(x) = P\left[z < \frac{\ln x - \mu}{\sigma}\right] \quad ; z_\alpha = \frac{\ln x - \omega}{\rho}$$

$$f(x) = \frac{1}{\rho x} f(z)$$

$$\mu = e^{\omega + \frac{1}{2}\rho^2} \quad ; \sigma^2 = e^{2\omega + \rho^2}\left[e^{\rho^2} - 1\right] = \mu^2\left[e^{\rho^2} - 1\right]$$

D. MAINTAINABILITY APPORTIONMENT (SEE NOTES)

E. MAINTAINABILITY DEMONSTRATION (SEE NOTES)

XII. SYSTEM EFFECTIVENESS

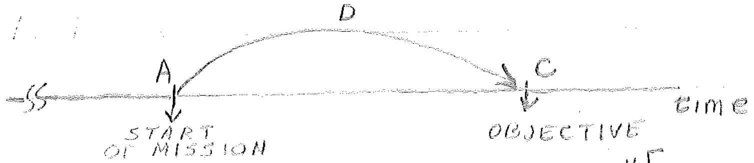
$$[\;E\;] = [A][D][C]$$

A = AVAILABILITY

C = CAPABILITY

D = DEPENDABILITY

E = SYSTEM EFFECTIVENESS



$$\#\text{SYSTEMS}\; n_{SYS}\left[\underset{\#\text{OBJECTIVES}\;n_o}{E}\right] = n_{SYSTEMS}\left[\underset{\#\text{STATES}\;n_{ST}}{A}\right]\#\text{STATES}\left[\underset{\#\text{STATES}\;n_{ST}}{D}\right]\#\text{STATES}\left[\underset{\#\text{OBJECTIVES}\;n_o}{C}\right]$$

A. AVAILABILITY

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n_{ST}} \\ a_{21} & & & \vdots & & \vdots \\ \vdots & & & & & \\ a_{i1} & \cdots & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & & \vdots & & \vdots \\ a_{n_{SY}} & & & a_{in_{SY}} & \cdots & a_{n_{SY}n_{ST}} \end{bmatrix}$$

$a_{ij} = P$[SYSTEM j WILL BE IN STATE i AT START OF MISSION]

EX: COMPUTE FROM $A = \frac{MTTF}{MTTF + MTTR}$

B. DEPENDABILITY

$$[D] = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1j} & \cdots & d_{1n_{ST}} \\ d_{21} & & & \vdots & & \vdots \\ d_{i1} & \cdots & \cdots & d_{ij} & \cdots & d_{in_{ST}} \\ d_{n_{ST}1} & \cdots & \cdots & d_{n_{ST}i} & \cdots & d_{n_{ST}n_{ST}} \end{bmatrix}$$

$d_{ij} = P$[END IN STATE OF REPAIR i GIVEN IT STARTED IN STATE j]

EX: COMPUTE R[MISSION EXECUTION TIME]

C. CAPABILITY

$$[C] = \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n_o} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in_o} \\ \vdots & & \vdots & & \vdots \\ c_{n_{ST}1} & \cdots & c_{n_{ST}j} & \cdots & c_{n_{ST}n_o} \end{bmatrix}$$

$c_{ij} = P$[ACCOMPLISHING OBJECTIVE i GIVEN SYSTEM IN STATE j]

(SHOULD BE GIVEN)

CHARACTERISTICS AND APPLICATIONS OF COMMON DISCRETE DENSITY FUNCTIONS

DENSITY	CHARACTERISTICS	APPLICATIONS
BINOMIAL	Only N "Occurrences" possible out of N Trials. Each trial must have the same probability of event occurrence as all other trials. There must be only two possible outcomes to each trial.	The number of failures in a Time Terminated, Non-Replacement Test. The number of sixes in 5 rolls of a die (or a single roll of 5 dice).
POISSON	An Infinite Number of "Occurrences" is possible. It is impossible to count the number of Non-Occurrences. The average rate of Occurrence must remain constant.	The number of failures in a Replacement Test of items obeying the Exponential Density. The number of meteorites hitting a satellite during one month.
	As an approximation to the Binomial, The probability of Occurrence must be small and N large.	The number of duds in 100,000 rounds of ammunition.

CHARACTERISTICS AND APPLICATIONS OF COMMON CONTINUOUS DENSITY FUNCTIONS

DENSITY	CHARACTERISTICS	APPLICATIONS
EXPONENTIAL (WEIBULL, $\beta = 1.0$)	Constant Hazard-items just as good at any age. Failures due to Random Causes. For single items where there is no heating, moving, flexing. For assemblies when they are complex.	Tires (Failure due to puncture). Digital Computers.
NORMAL (WEIBULL, $\beta > 1.0$) ONLY APPROXIMATE	Increasing Hazard-Wearout. Failures generally attributable to a single cause. For single items where there is heating, moving, flexing.	Tires (Failure due to tread wear). Bearings. Fan Belts.
	A good maintenance time model for simple tasks with no opportunity for varying the method.	Depot repair tasks.
WEIBULL $\beta < 1$	Decreasing Hazard-Infant Mortality-Items improve with age. May be due to a poorly controlled manufacturing process or an inability to inspect adequately.	Solid State Devices. Diaphragm Fuel Pumps
LOG-NORMAL	Maintenance Time Model for a complex system. Maintenance Time Mode for single tasks where there is an opportunity for varying the method.	C 130 E Aircraft, flight line maintenance. Spark Plug Removal and replacement.

TITLES & ADDRESSES OF MILITARY MATERIAL COVERED

1. H-108 4/29/60

QUALITY CONTROL AND RELIABILITY HANDBOOK (ITERIM)

"SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY TESTING (BASED ON EXPONENTIAL DISTRIBUTION)"

OFFICE OF THE SECRETARY OF DEFENSE
(SUPPLY & LOGISTICS) WASHINGTON 25, D.C.

2. TR 3 9/30/61

QUALITY CONTROL AND RELIABILITY: TECHNICAL REPORT

"SAMPLING PROCEDURES AND TABLES FOR LIFE AND RELIABILITY BASED ON THE WEIBULL DISTRIBUTION (MEAN LIFE CRITERION)"

OFFICE OF THE ASSISTANT SECRETARY OF DEFENSE
(INSTALLATIONS AND LOGISTICS), WASHINGTON 25, D.C

3. TR 4 2/28/62

-SAME AS ABOVE-

" — (HAZARD RATE CRITERION)"

4. TR-6

-SAME AS ABOVE- 2/15/63

" — (RELIABLE LIFE CRITERION)"

5. MIL-STD-414, 11 JUNE 57

SUPERSEDING ORD-M608-10, JUN 1954, NAVORD OSTD 80, 8 MAY 72

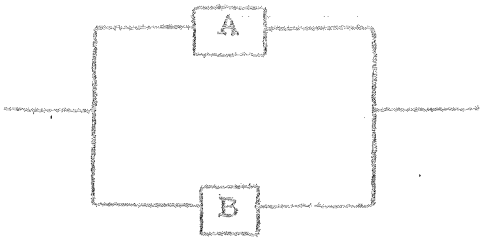
"MILITARY STANDARD-SAMPLING PROCEDURES AND TABLES FOR INSPECTION BY VARIABLES FOR PERCENT DEFECTIVE"

OFFICE OF THE SECRETARY OF DEFENSE
WASHINGTON 25, D.C.

6, MIL-STD-105D 29 APRIL 63
SUPERSEDING MIL-STD 105C 18 JULY 61
"MILITARY STANDARD-SAMPLING PROCEDURES AND
TABLES FOR INSPECTION BY ATTRIBUTES"
DEPT. OF DEFENSE, WASHINGTON D.C.

ELEMENTS OF RELIABILITY AND MAINTAINABILITY
Problem Set #1

1. A system is described by the following reliability block diagram:



Subsystem A has Weibull distributed failure times with $\eta = 15$ hours, $\beta = 2.1$, and $\gamma = 0$.

Subsystem B has normally distributed failure times with $\mu = 18$ hours and $\sigma = 6$ hours.

If subsystem A is new, and subsystem B has been used for 6 hours without failure, what is the reliability for a 15-hour mission? (Assume failures of A and B are independent.)

2. Test results show the time to failure of the fuel system for a given vehicle is distributed in accordance with a Weibull Distribution with $\beta = 2$ and $\eta = 100$. There are three potential users for this fuel system and they have stated their reliability requirements as follows:

   a. Probability of completing a twenty-hour mission without failure must be at least .95.

   b. The mean time to failure for the fuel system is to be no less than 100 hours.

   c. The hazard rate at age fifty hours must not exceed 80 failures per hour.

   Will the fuel system meet the reliability requirements for all three potential users? (Show numerical comparisons.)

3. Solve either problem a or b (below) and explain why you chose not to solve the other.

   a. Item A fails in accordance with the normal distribution: $\mu = 50$, $\sigma = 25$. If five of these items are tested for 40 hours each, with replacement, what is the probability of exactly two failures?

   b. Item B fails in accordance with the exponential distribution, $\theta = 90$. If five of these items are tested for 30 hours without replacement, what is the probability of exactly two failures?

4. A parachute system is being developed which employs a primary and a secondary (emergency) chute. The system is designed so that deployment of the second chute releases the first chute to prevent entanglement. Based on testing and previous knowledge, the following information is available:

   a. The probability of the primary chute operating correctly is .9.

   b. The probability of the secondary chute operating correctly is .8.

   c. There is a .1 probability that upon failure of the first chute, the jumper will panic and fail to deploy the second.

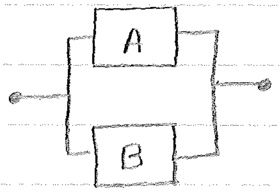
   d. There is a .2 probability that the jumper will deploy the second chute in spite of the first operating successfully.

   Draw the block diagram and compute the predicted probability of this parachute system operating successfully.

5. Consider a system design comprised of two subsystems A and B in series. A and B are in process of development and developmental testing has shown MTTF to be 100 and 500 hours, respectively. (Assume exponential failure times.) The required system reliability level is expressed as a hazard rate of .008 failures per hour. Using the ARINC apportionment technique, what MTTF goals should be established for subsystems A and B?

40/40

1.



a. FIRST, FIND THE RELIABILITY OF SUBSYSTEM B (GIVEN ITS HISTORY) FROM 6 TO 15 HRS GIVEN $\mu = 18$ HRS & $\sigma = 6$ HRS

$$R_B' = P[X > 21 / X > 6] = \frac{R_B(21)}{R_B(6)}$$

$$\rightarrow R_B(21) = P[X > 21]$$

$$= P\left[Z = \frac{X-\mu}{\sigma} > \frac{21-18}{6} = \frac{1}{2}\right]$$

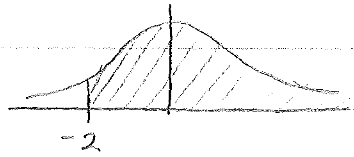
FROM TABLE:

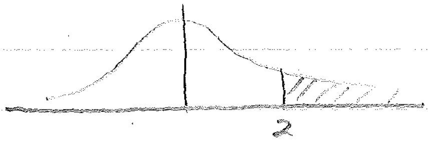
$$P[Z > 0.5] = 0.30854$$

THUS: $R_B[21] = 0.30854$ ✓

$$\rightarrow R_B(6] = P[X > 6]$$

$$= P\left[Z > \frac{6-18}{6} = -2\right]$$





$$P[Z > -2] = 1 - P[Z > 2]$$

$$= 1 - 0.02275$$

$$= 0.97725$$

THUS $R_B(6) = 0.97725$ ✓

∴ THE RELIABILITY OF B FOR THE MISSION IS THUS

$$R_B' = \frac{0.30854}{0.97725} = 0.31572$$ ✓

b. FOR SYSTEM A, WHICH IS NEW AND WEIBULLY DISTRIBUTED:

$$R_A(x) = e^{-(\frac{x}{\eta})^\beta}$$

GIVEN

$x = 15$ HR

$\beta = 2.1$

$\eta = 15$ HRS

WE HAVE

$$R_A(15) = e^{-(\frac{15}{15})^{2.1}} = e^{-1} = 0.36788$$ ✓

c. FOR SIMPLE ACTIVE REDUNDANCY

$$1 - R_S = Q_S = \prod_{i=1}^{n} Q_i$$

THUS, FOR THE SYSTEM IN QUESTION

$$Q_S = (1 - R_B')(1 - R_A)$$
$$= (1 - 0.31572)(1 - 0.36788)$$
$$= (0.68428)(0.63212)$$
$$= 0.43255$$

$$\therefore R_S = 1 - Q_S$$
$$= 1 - 0.43255$$
$$= 0.56745$$ ✓

2. WEIBULL DISTRIBUTION

$\beta = 2$; $\eta = 100$ HRS

a. IS $R(24 \text{ HRS}) \geq 0.95$?

FOR WEIBULL

$$R(x) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$$

THUS

$$R(24) = e^{-\left(\frac{24}{100}\right)^{2}}$$

$$= 0.94403$$ OK ✓

$\therefore$ THE RELIABILITY SPECIFICATION $R(24) > 0.95$ IS **NOT** MET

b. THE MEAN OF A WEIBULL DISTRIBUTION IS

$$\mu = \eta \Gamma\left(\frac{1}{\beta} + 1\right)$$

ERGO

$$\mu = 100 \; \Gamma(1.5)$$

FROM TABLE : $\Gamma(1.5) = 0.88623$

HENCE, THE MEAN IS

$$\mu = 88.623 \text{ HRS}$$

$\therefore$ THE MEAN SPECIFICATION $\mu = MTBF > 100$ IS **NOT** MET ✓

c. THE HAZARD RATE FOR THE WEIBULL IS

$$h(x) = \left(\frac{\beta}{\eta}\right)\left(\frac{x}{\eta}\right)^{\beta - 1}$$

AT $x = 50$, $h(50) = \left(\frac{2}{100}\right)\left(\frac{50}{100}\right)^{2-1}$

$$= \frac{1}{100} = 0.01 \; \frac{\text{FAILURES}}{\text{HR}}$$

$\therefore$ THE HAZARD RATE SPECIFICATION $[h(x) < 80]$ IS (GENEROUSLY) MET. ✓

3b. EXPONENTIAL DISTRIBUTION

$\theta = \frac{1}{\lambda} = 90$ HRS

$n = 5$ ITEMS $\quad T = 30$ HRS

FIND $P[\text{2 FAILURES}] = P[\underline{X} = 3] \ni \underline{X} = \#$SUCCESSES

FIRST, WE GOTTA FIND $R(30 \text{ HRS})$. GENERALLY

$R(x) = e^{-\lambda x}$

THUS $R(30) = e^{-\frac{30}{90}} = e^{-\frac{1}{3}} = 0.71653$

THE SUCCESS PROBABILITY IS GIVEN THROUGH THE BINOMIAL DISTRIBUTION:

$$P[\underline{X} = x] = \binom{n}{x} p^x q^{n-x}$$

FOR PROBLEM AT HAND:

$n = 5$

$x = 3$

$p = R(30) = e^{-\frac{1}{3}} = 0.716.53$

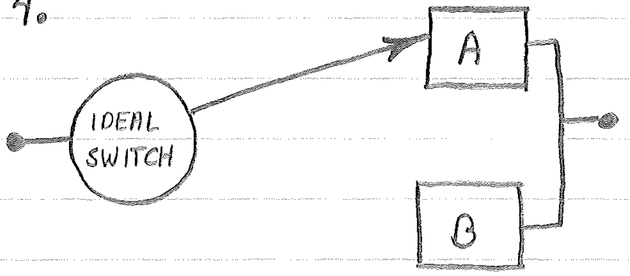
$q = 1 - R(30) = 1 - e^{-\frac{1}{3}} = 0.28347$

$$\therefore P[\underline{X} = 3] = \underset{\searrow 10}{\binom{5}{3}} (0.71653)^3 (0.28347)^2$$

$$= 0.29561 \checkmark$$

THE PROBLEM WITH PART "a" IS THE PHRASE "WITH REPLACEMENT". IF THE POPULATION WAS EXPONENTIALLY DISTRIBUTED, A POISSON EXAMINATION OF THE QUESTION WOULD BE IN ORDER. ✓ BUT FOR A NORMAL, WE USE WHAT(?)

4.



A: PRIMARY CHUTE WORKS OKAY

B: SECONDARY CHUTE WORKS OK.

$P[A] = p_A = 0.9$

$P[B] = p_B = 0.8$

$q_w = P[\text{FAILURE TO DEPLOY / "A" HAS FAILED}] = 0.1$

$q_{w'} = P[\text{PREMATURE DEPLOYMENT}] = 0.2$

THE RELIABILITY FOR SUCH A SYSTEM, AS DERIVED IN CLASS VIA TREE DIAGRAM, IS

$R_S = P_A P_W' + P_A q_{w'} P_B + q_A P_W P_B$

WHERE $q = 1 - p$

THUS

$R_S = (0.9)(0.8) + (0.9)(0.2)(0.8) + (0.1)(0.9)(0.8)$

$= 0.72 + 0.144 + 0.072$

$= 0.936$ ✓

IN TEXT: (pg III-17)

$R_S = 1 - (P_A q_{w'} q_B + q_A q_W + q_A P_W q_B)$

$= 1 - [(0.9)(0.2)(0.2) + (0.1)(0.1) + (0.1)(0.9)(0.2)]$

$= 1 - [0.036 + 0.010 + 0.018]$

$= 1 - 0.064$

$= 0.936$ ✓ (SAME THING)

PROBLEM SET #2

Elements of Reliability and Maintainability

1. Fifteen experimental grenades have been tested. Two of them failed to fire properly.

   a. State a point estimate of the reliability of this type of grenade.

   b. State a lower 95% confidence limit for the reliability of this type of grenade.

   c. What are three reasons for the 95% lower confidence limit being so low?

2. Nine aircraft alternators were placed on test without replacement until they all failed. The resultant failure times (in hours) were as follows:

   2400 4000 7200 10,000 14,200 17,200 26,000 34,200 43,000

   a. Using the graph paper provided, determine if the Weibull distribution provides a reasonable failure model; and, if it does, estimate the parameters, η and β.

   b. Two procedures have been proposed to improve the reliability of these alternators in service. One is to run-in each alternator under load for 2000 hours prior to installation to screen out the inferior ones. The other procedure is to time replace them after they have accumulated 18,000 hours of operation. Comment on these two proposals.

3. Nine items were life tested with replacement until the occurrence of the fourth failure. The resulting failure times were 3, 15, 29, and 42 hours.

   a. Assuming the exponential density for failure times, find point estimates for the mean time to failure and the reliability for an 18-hour mission.

   b. Also find an 80% confidence interval for the hazard rate and a 90% lower confidence limit for R(18).

4. A sample of three items was placed on life test until all items failed. The resultant failure times were 50, 100, and 150 hours.

   Use a goodness-of-fit test to determine whether the assumption of a constant hazard rate is reasonable. Use a level of significance of 0.2 or less.

5. A newly developed radio receiver is to be tested to determine that its failure rate is sufficiently low. If the failure rate is .0001 failures per hour, there should be only a 5% chance of failing the test. On the other hand, if the failure rate is .0002 failures per hour, there should be a 90% probability of failing the test.

   a. Assuming that the failure rate is constant, how many radios should be tested for 1000 hours each if they are repaired as necessary during the test?

   b. How many failures are permissible for acceptance?

   c. If the test is passed, what confidence would we have that the failure rate is less than .0001 failures per hour?

-6

1. $n = 15 \quad r = 2 \Rightarrow d = 13$

   a. $\hat{R} = \frac{d}{n} = \frac{13}{15} = 0.8667$

   b. USING TABLE K_2

      ENTER WITH $r = 2, n = 15$;

      $P[X \geq 2] = 1 - 0.363$

      $= 0.637 \Leftarrow$ 95% CONFIDENT

   c. THE LOWER CONFIDENCE LIMIT WOULD BE LARGER IF

      1. WE HAD TESTED MORE ITEMS (ie INCREASED n)
      2. WE HAD OBSERVED LESS FAILURES (ie DECREASE r)
      3. WE HAD DEMANDED LESS CONFIDENCE (ie INCREASE α)

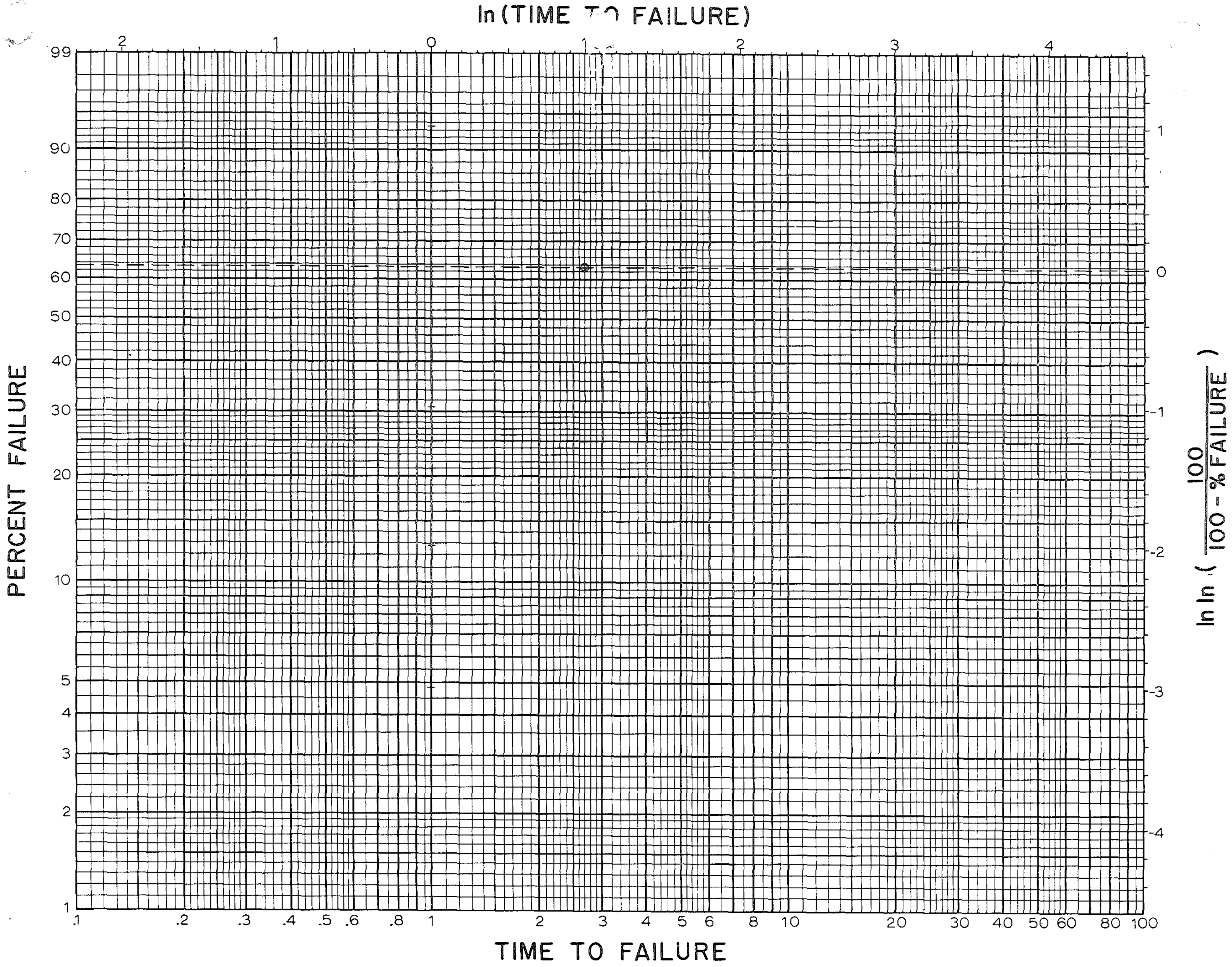
K+E PROBABILITY X 90 DIVISIONS 46 8000
MADE IN U.S.A.
KEUFFEL & ESSER CO.

99.99 99.9 99.8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0.5 0.2 0.1 0.05 0.01

0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99

K+E LOGARITHMIC
3 X 3 CYCLES
KEUFFEL & ESSER CO.

46 7400
MADE IN U.S.A.


ln (TIME TO FAILURE)
2
1
0
1
2
3
4
PERCENT FAILURE
99
90
80
70
60
50
40
30
20
10
5
4
3
2
1
.1
.2
.3
.4
.5
.6
.8
1
2
3
4
5
6
8
10
20
30
40
50
60
80
100
TIME TO FAILURE
ln ln (100 / 100 - % FAILURE)
1
0
-1
-2
-3
-4

ARMY MANAGEMENT ENGINEERING TRAINING AGENCY

USAMETA CLASS NUMBER 053 PAGE NUMBER 003

FROM 12-02-74 TO 12-20-74 COURSE TITLE ELMS OF RELIABILITY & MAINT

COURSE INSTRUCTOR CARY CHITTENDEN ROOM 01

NR	GRADE/RANK	NAME	POSITION OR TITLE	SERVICE	INSTALLATION	
01		BUCK, CHRISTINE M	MATHEMATICIAN	NAVY	NAVAL SEA SYSTEMS [illegible] WASH	DC
02		CANTRELL, MICHAEL R	CHEMICAL ENGINEER	AMC ARMCOM	USA ARMAMENT COM [illegible]	IL
03		COWAN, MARION C	MATH STATISTICIAN	AMC TECOM	USA ARCTIC TEST [illegible] ILE	WA
04		DAVIS, WILLIAM J	ELECTRONICS ENGINEER	AF	MCCHORD AIR FORCE BASE	WA
05		EILBERT, CRAIG F	EQUIPMENT SPECIALIST	USASA	USASAMSC VHF WARRENTON	VA
06		FORTUNE, JOHN D	EQUIPMENT SPECIALIST	AMC ARMCOM	FRANKFORD ARS PHILADELPHIA	PA
07		HANES, HAL D	TEST OFFICER	AMC TACOM	PROJECT MGR MICV WARREN	MI
08		LUTHER, THOMAS J	PROJECT ENGINEER	AF	AFPRO ST LOUIS	MO
09		MACLEOD, CHARLES A	GENERAL ENGINEER	AMC ARMCOM	USA ARMAMENT COM [illegible]	IL
10		MARKS, ROBERT J II	ELECTRONICS ENGINEER	NAVY	NAVAL AMMUNITION [illegible] CRANE	IN
11		PIHL, ROBERT L	QUAL ASSUR ENGINEER	AF	EGLIN AIR FORCE BASE	FL
12		RAHM, RICHARD O	QUAL ASSUR ENGINEER	DSA	DCASR ATLANTA	GA
13		ROCKWELL, ROBERT H	QUALITY ASSUR SPEC	DSA	DEFENSE SUP AGCY ALEXANDRIA	VA
14		ROMSOS, ARDEN E	ELEC ENGINEER	MARINE CORPS	HQMC WASHINGTON	DC
15		SANDERS, JERRELL L	TECHNICAL ADVISOR	AMC TECOM	USA ARCTIC TEST [illegible] SEATTLE	WA
16		SHOR, CONRAD	MICROWAVE ENGINEER	AMC ARMCOM	FRANKFORD ARS PHILADELPHIA	PA
17		SMITH, OMER L	PROJECT ENGINEER	AF	HILL AIR FORCE BASE	UT
18		SMITH, RAYMOND B	AEROSPACE ENGINEER	AMC AVSCOM	USA AVN EGR FLT [illegible] EDWARDS	CA
19		WHITFILL, PATRICK J	ELECTRONIC ENGINEER	AMC ECOM	USAECON FORT MONMOUTH	NJ
20		WINANS, F N	MISSILE SYSTEMS EGR	AMC TECOM	WHITE SANDS MISSILE RANGE	NM
21		WOND, ALAN N	QA MECHANICAL EGR	AF	EGLIN AIR FORCE BASE	FL
22		WOOLARD, JAMES L	ELEC ENGINEER	TRADOC	USALOGC FORT LEE	VA
23		WOYTEK, ARTHUR H	CH MAINT EVALUATION	AMC TECOM	USAATC APO SEATTLE	WA
24		WRIGHT, LEON W	AEROSPACE ENGINEER	AF	KIRTLAND AIR FORCE BASE	NM

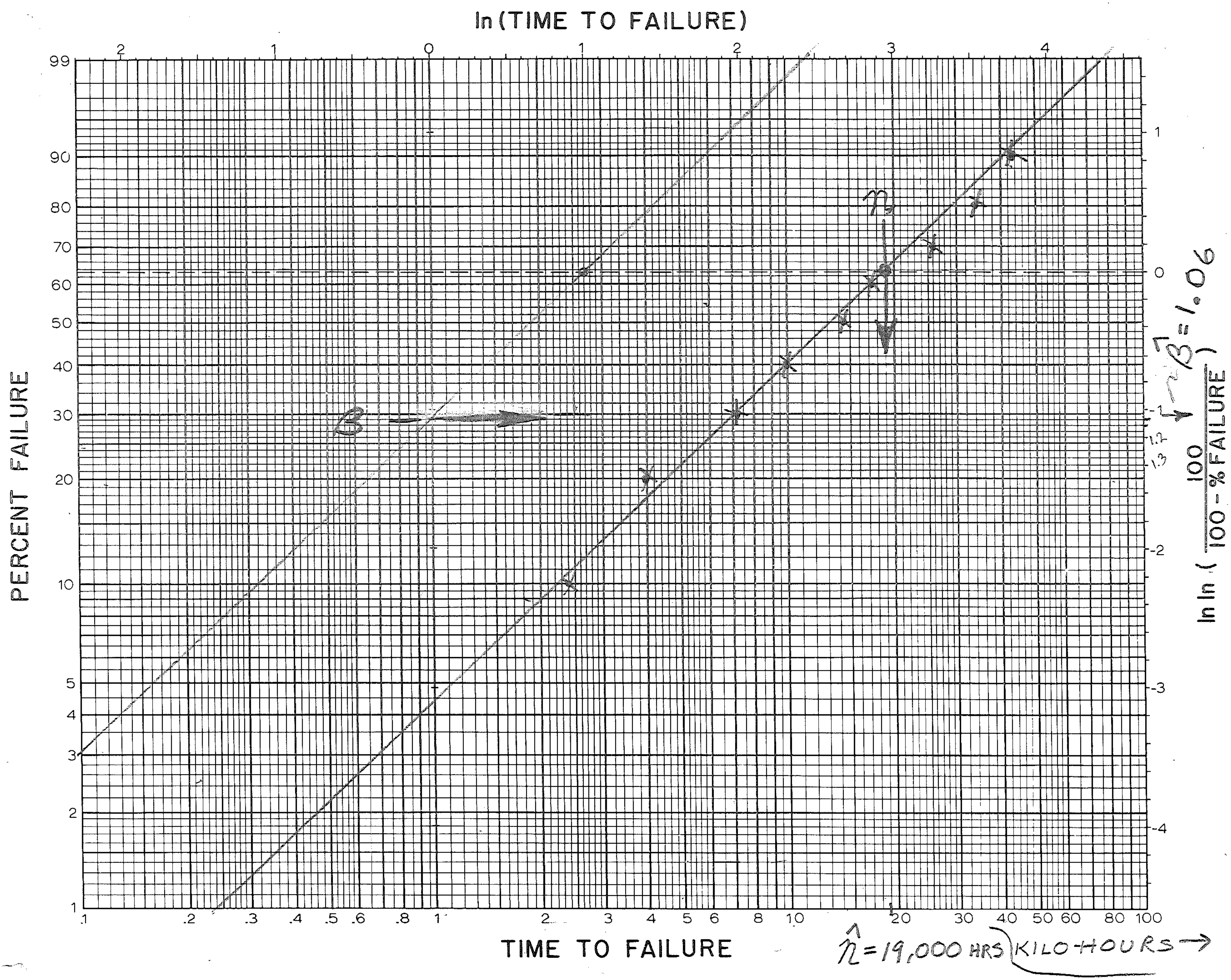
SUMMARY CIVILIAN 014 MILITARY 010 TOTAL 024 FOREIGN NATIONAL 000 REIMBURSABLE 000

2. $n = 9$

a. i	$\frac{100i}{n+1} = 10i$	X_f
1	10	2400
2	20	4000
3	30	7200
4	40	10,000
5	50	14,200
6	60	17,200
7	70	26,000
8	80	34,200
9	90	43,000

FROM CHART:

$\hat{\eta} = 19{,}000$ HRS ; $\hat{\beta} = 1.06$


ln (TIME TO FAILURE)
2
1
0
1
2
3
4
99
90
80
70
60
50
40
30
20
10
5
4
3
2
1
PERCENT FAILURE
1
0
-1
1.2
1.3
-2
-3
-4
ln ln (100 / 100 - % FAILURE)
.1
.2
.3
.4
.5
.6
.8
1
2
3
4
5
6
8
10
20
30
40
50
60
80
100
TIME TO FAILURE
β
η
β = 1.06
$\hat{\eta}$ = 19,000 HRS
KILO-HOURS →

b. PROPOSAL 1

BURN IN TEST FOR 2000 HOURS PRIOR TO INSTALLATION

PROPOSAL 2

REPLACE EVERY 18,000 HRS.

COMMENTS:

THE ESTIMATED β IS VERY NEARLY EQUAL TO UNITY, SUGGESTING THE SAMPLED POPULATION IS NEARLY EXPONENTIAL WITH AN MTTF $= \hat{\theta} \simeq \eta = 19{,}000$ HRS. FOR THE CONSTANT HAZARD RATE $(= \frac{1}{\theta})$ OF THE EXPONENTIAL, A BURN IN TEST IS NOT ADVISABLE SINCE EXPONENTIAL POPULATION MEMBERS DO NOT AGE. REPLACEMENT OF THE UNIT PRIOR TO THE ESTIMATED MTTF IS THE MORE ADVISABLE PROPOSAL.

-1 THEY STILL DO NOT AGE

3. $n = 9$

FTT WITH REPLACEMENT : $r = 4$

i	x_i
1	3
2	15
3	29
4	42

a. EXPONENTIAL ASSUMPTION

FOR FTT WITH REPLACEMENT:

$$X_t = n\,X_r = (9)(42)$$

$$\hat{\Theta} = \frac{X_t}{r} = \frac{(9)(42)}{4} = 94.5 \text{ HRS}$$

b. i. FIND 80% CONF. LIMIT FOR HAZARD RATE. $\Rightarrow \alpha = 0.2$

FOR EXPONENTIAL:

$$h(t) = \lambda = \frac{1}{\Theta}$$

$\Rightarrow$ FIND AN 80% CONF. LIMIT FOR Θ :

FOR EXPONENTIAL FTT :

$$\frac{2r\hat{\Theta}}{\chi^2_{\alpha/2;2r}} \leq \Theta \leq \frac{2r\hat{\Theta}}{\chi^2_{1-\alpha/2;2r}} \quad ; \frac{\alpha}{2} = 0.1, r = 4$$

~~$\frac{2(4)(94.5)}{7.779} \leq \Theta \leq \frac{2(4)(94.5)}{0.297}$~~

~~$97.185 \leq \Theta \leq 2545.45$~~

~~OR SINCE $\lambda = \frac{1}{\Theta}$:~~

~~$\frac{1}{2545.45} \leq \lambda \leq \frac{1}{97.185}$~~

~~$0.000393 \leq \lambda \leq 0.0103 \Leftarrow 80\%$ CONFIDENCE~~

$$\chi^2_{\alpha/2;2r} = \chi^2_{0.1;8} = 13.362$$
$$\chi^2_{1-\frac{\alpha}{2};2r} = \chi^2_{0.9;8} = 3.490$$

80% CONFIDENT THAT

$$\frac{(2)(4)(94.5)}{13.362} \leq \theta \leq \frac{(2)(4)(94.5)}{3.490}$$

$$56.578 \leq \theta \leq 216.62$$

$\theta = \frac{1}{\lambda} \Rightarrow$ 80% CONFIDENT THAT

$$\frac{1}{216.62} \leq \lambda \leq \frac{1}{56.578}$$

$$0.00462 \leq \lambda \leq 0.0177$$

ii. FIND A 90% LOWER CONFIDENCE INTERVAL FOR $R(18)$.

WE KNOW FROM ABOVE THAT

$\theta \geq 56.578$

IS 90% CONFIDENT.

THUS:

$$R(t) \geq e^{-\frac{x}{56.578}}$$
$$R(18) \geq e^{-\frac{18}{56.578}} = 0.7275$$

IS 90% CONFIDENT

4. $n = 3$

SINCE WE HAVE A SMALL SAMPLE SIZE, WE SHOULD EMPLOY KOLMOGOROV-SMIRNOV TEST.

[CONSTANT HAZARD ⇒ EXPONENTIAL]

$$\hat{\Theta} = \frac{1}{3}\sum_{i=1}^{3} x_i = \frac{50+100+150}{3} = 100$$

-2

$F(x) = 1 - e^{-x/\theta}$

i	x_i	$\hat{F}(x_i)$	$F(x_i) = e^{-\frac{x_i}{100}}$	$d_i = \lvert F(x_i) - \hat{F}(x_i) \rvert$
1	50	$\frac{1}{3}$	1-0.6065	0.2732
2	100	$\frac{2}{3}$	1-0.3679	0.2988
3	150	1	1-0.2231	0.7769

$d_{i\,MAX} = d_3 = 0.7769$

WE REJECT EXPONENTIAL ASSUMPTION IF

$d > d_{\alpha;n} = d_{0.2;3}$

FROM TABLE J : $d_{0.2;3} = 0.565$

$0.7769 > 0.565$

∴ WE REJECT ASSUMTION OF CONSTANT HAZARD RATE

5. $H_0 : \lambda = \lambda_0 = 10^{-4} \frac{\text{FAIL}}{\text{HRS}}$ $\alpha = 0.05$

$H_1 : \lambda = \lambda_1 = 2 \times 10^{-4} \frac{\text{FAIL}}{\text{HRS}}$ $\beta = 0.10$

EQUIVALENT TO

$H_0 : \Theta = \Theta_0 = 10^4$ HRS $\alpha = 0.05$

$H_1 : \Theta = \Theta_1 = 0.5 \times 10^4$ HRS $\beta = 0.10$

a, b. GIVEN TTT WITH $T = 10^3$ HRS

(TEST WITH REPLACEMENT)

FIND n AND r_0 :

n AND r_0 MUST SATISFY

$$1 - \alpha = \sum_{K=0}^{r_0 - 1} \frac{1}{K!} \left(\frac{nT}{\Theta_0}\right)^K e^{-\left(\frac{nT}{\Theta_0}\right)}$$

$$\beta = \sum_{K=0}^{r_0 - 1} \frac{1}{K!} \left(\frac{nT}{\Theta_1}\right)^K e^{-\left(\frac{nT}{\Theta_0}\right)}$$

-2 USING H-108

$\frac{\Theta_1}{\Theta_0} = \frac{1}{2}$; $\frac{T}{\Theta_0} = \frac{10^3}{10^4} = \frac{1}{10}$

FROM H-108, Pg 2.53, TABLE 2C-4

$n = 124$, $r_0 = 19$

USING THORNDIKE CHART

$n = 124$, $r_0 = 19$

$\frac{nT}{\Theta_0} = \frac{(124)10^3}{10^4} = 12.4$

WITH $1 - \alpha = 0.95$, WE GET $r_0 - 1 = 18$ GOOD

$\frac{nT}{\Theta_1} = \frac{(124)10^3}{5 \times 10^3} = 24.8$

WITH $\beta = 0.10$ WE GET $r_0 - 1 = 18$ GOOD

~~c. FIND $P[\lambda \leq 10^{-4} \frac{\text{FAIL}}{\text{HR}} / H_0 \text{ IS ACCEPTED}]$~~

~~$= P[\Theta > 10^4 \text{ HRS} / H_0 \text{ IS ACCEPTED}]$~~

~~$1 - \alpha = P[\text{ACCEPTING } H_0 / H_0 \text{ IS CORRECT}]$~~

~~$= P[\text{ACCEPTING } H_0 / \Theta > 10^4 \text{ HRS}]$~~

C. FIND $P[H_0 \text{ IS TRUE} / H_0 \text{ IS ACCEPTED}]$

$A = P[H_0 \text{ IS ACC}]$

$T = P[H_0 \text{ IS TRUE}]$

$P[T/A]\,P[A] = P[T]\,P[A/T]$

$$P[T/A] = \frac{P[T]}{P[A]} \times P[A/T]$$

$P[A/T] = 1-\alpha = P[H_0 \text{ ACC} / H_0 \text{ TRUE}]$

$$\Rightarrow P[T/A] = (1-\alpha)\,\frac{P[T]}{P[A]}$$

$P[A] = P[r = \#\text{OBSERVED FAILURES} < r_0 = 19]$

$= P[r < r_0]$

$$\Rightarrow P[T/A] = \frac{(1-\alpha)}{P[r<r_0]} \times P[T]$$

$$P[r<r_0] = \sum_{k=0}^{r_0-1} \frac{1}{k!}\left(\frac{nT}{\theta}\right)^k e^{-\left(\frac{nT}{\theta}\right)}$$

$P[T] = P[\theta > 10^4 \text{ HRS}]$

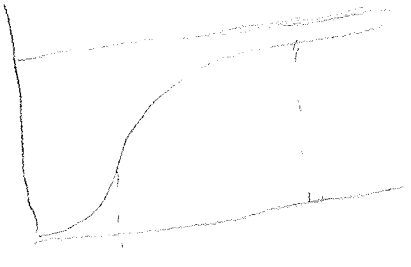
$$\Rightarrow P[T/A] = \frac{(1-\alpha)\,P[\theta > 10^4 \text{ HRS}]}{P[r<r_0]}$$

$5\% = \alpha = 1-P_a$ IF TEST IS PASSED

-1

$\alpha = P[\bar{A}/T]$

$1-\alpha$



ERM PROBLEM SET

1. A system fails at a constant rate of .01 failures/hour. Active repair times obey a log-normal distribution with $\overset{\omega}{\mu_{\ln x}}$ = 1.2 in minutes and $\overset{\rho}{\sigma_{\ln x}}$ = .8 in minutes. Delay time obeys a normal distribution with μ = 150 minutes and σ = 50 minutes. Determine inherent and operational availability of this system if there is no preventive maintenance required.

2. Using H-108, find a life test sampling plan which will accept, with probability .95, a lot of components which has a mean life of 700 hours. The test is to be terminated at the occurrence of the fourth failure. Twelve components are to be placed on test. Use of this plan resulted in the following failure times:

44	188	240	576

Should this lot be accepted?

3. Torsional strengths of the human tibia (the leg bone often broken in skiing) are approximately normally distributed, with a mean of 1000 kg cm and a standard deviation of 200 kg cm. If the skiing population used safety bindings with release torques that are approximately normally distributed, with a mean of 400 kg cm and a standard deviation of 10 kg cm, what is the probability that a potential leg-breaking spill will cause a broken leg? (Consider just one leg and assume that the torque setting is independent of the individual skier.)

4. Tomorrow you will be leaving the Quint-Cities aboard a four-engine aircraft. You have found that the probability of landing safely depends on the number of engines operating at the time of landing. These probabilities are: .999 with four engines, .859 with three engines, .401 with two engines, and 0 with one or fewer engines. Realizing the implications involved, you contacted the airlines and found that the mean life of these engines is 24,000 air miles and that failures occur randomly. Contacting the FAA, you found that takeoff is only permitted with all four engines operating.

a. Using this information, set up the availability (at the time of takeoff), dependability, and capability matrices which would be associated with a 1200-mile flight. (Put numerical entries in the matrices, but multiplication need not be carried out.)

b. If the A, D, and C matrices were to be multiplied, what meaning would you attach to the resultant single number for system effectiveness?

(The probabilities given here are for illustrative purposes only and are not meant to be realistic.)